Finding Turning Points

Starter

1. **(Review of last lesson)** Solve $x^2 - 2x - 7 = 0$ by completing the square, giving your answer exactly.

Working: $x^2 - 2x - 7 \equiv (x-1)^2 - (-1)^2 - 7 \equiv (x-1)^2 - 8$

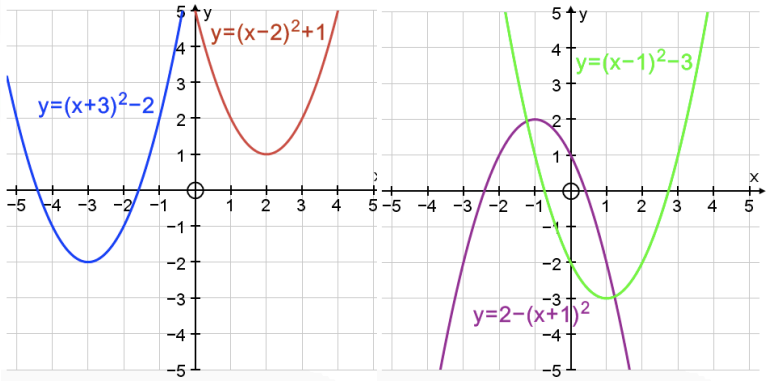
$$(x-1)^2 - 8 = 0$$
$$(x-1)^2 = 8$$
$$x - 1 = \pm\sqrt{8}$$
$$x - 1 = \pm 2\sqrt{2} \qquad \textit{since } \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$
$$x = 1 \pm 2\sqrt{2}$$

2. **(Review of last lesson)** Express $2x^2 - 16x - 7$ in completed square form.

Working:
$$\begin{aligned} 2x^2 - 16x - 7 &\equiv 2[x^2 - 8x] - 7 \\ &\equiv 2[(x-4)^2 - 4^2] - 7 \\ &\equiv 2[(x-4)^2 - 16] - 7 \\ &\equiv 2(x-4)^2 - 32 - 7 \\ &\equiv 2(x-4)^2 - 39 \end{aligned}$$

E.g. 1 (a) By looking at the graphs below write down the coordinates of the vertex of the graph.

Completed square form	Vertex
$y = (x+3)^2 - 2$	
$y = (x-2)^2 + 1$	
$y = 2 - (x+1)^2$	
$y = (x-1)^2 - 3$	



(b) Hence conjecture where the coordinates of the curve $y = (x+p)^2 + q$ will be.

Working: (a)

Completed square form	Vertex
$y = (x-2)^2 + 1$	$(2, -1)$
$y = (x+3)^2 - 2$	$(-3, -2)$
$y = (x+1)^2 + 2$	$(-1, 2)$
$y = (x-1)^2 - 3$	$(1, -3)$

(b) $(-p,\ q)$

E.g. 2 Write down the coordinates of the turning point of these parabolas:

(a) $y = 7(x+1)^2 + 2$ (b) $y = (x-2)^2 + 3$

Working: (a) (−1, 2) (b) (2, 3)

E.g. 3 Find the turning point of the following parabolas:

(a) $y = x^2 + 20x + 500$ (b) $y = x^2 - 9x + 11$

Working: (a) $y = x^2 + 20x + 500 \equiv (x+10)^2 - 10^2 + 500 = (x+10)^2 + 400$
The turning point is at (−10, 400)

(b) $y = x^2 - 9x + 11 \equiv (x-4.5)^2 - (-4.5)^2 + 11 = (x-4.5)^2 - 9.25$
The turning point is at (4.5, −9.25)

E.g. 4 Write down a possible equation for a parabola given that the turning point is:

(a) (3, 4) (b) (−5, 1)

Working: (a) $y = k(x-3)^2 + 4$ where k is any number not equal to zero
(b) $y = k(x+5)^2 + 1$ where k is any number not equal to zero

E.g. 5 Write down (i) the line of symmetry and (ii) the maximum or minimum value of:

(a) $y = 8(x+6)^2 - 60$ (b) $y = 15 - 3(x-1)^2$

Working: (a) (i) Put the expression inside the bracket equal to zero and solve
$x + 6 = 0$
$x = -6$ is the equation of the line of symmetry

(ii) Put the bracket equal to zero: $y = 8 \times (0)^2 - 60 = -60$
Coefficient of x^2 is ***positive*** so the ***minimum*** value is −60

(b) (i) Put the expression inside the bracket equal to zero and solve
$x - 1 = 0$
$x = 1$ is the equation of the line of symmetry

(ii) Put the bracket equal to zero: $y = 15 - 3(0)^2 = 15$
Coefficient of x^2 is ***negative*** so the ***maximum*** value is 15

Video: Quadratic graphs (completing the square)
Video: Sketching quadratics

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p399 E12.5 Qu 1-5
A*-G class textbook:	No exercise available
9-1 homework book:	p399 E12.5 Qu 1-10
A*-G homework book:	No exercise available

Homework book answers (only available during a lockdown)