Fractional Indices when the Numerator is 1

Starter

1. **(Review of Y9 material)**

Simplify: (a) $3x^7 \times 5x^6$ (b) $\dfrac{14x^9}{2x^4}$ (c) $(2x)^3$ (d) $5x^{-2}$

Working: (a) $3x^7 \times 5x^6 = (3 \times 5)x^{7+6} = 15x^{13}$

(b) $\dfrac{14x^9}{2x^4} = 7x^{9-4} = 7x^5$

(c) $(2x)^3 = 2x \times 2x \times 2x = 8x^3$

(d) $5x^{-2} = \dfrac{5}{x^2}$ ***negative index (see below)***

2. **(Review of Y9 material)** State the value of: (a) 23^0 (b) $5a^0$

Working: (a) 1 ***zero index (see below)***

(b) $5a^0 = 5 \times 1 = 5$ ***zero index (see below)***

E.g. 1 Without using a calculator, find the value of:

(a) $16^{\frac{1}{4}}$ (b) $64^{\frac{1}{3}}$ (c) $125^{\frac{1}{3}}$

(d) $243^{\frac{1}{5}}$ (e) $729^{\frac{1}{6}}$ (f) $1296^{\frac{1}{4}}$

Working: (a) $16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2^{4\times\frac{1}{4}} = 2^1 = 2$

(b) $64^{\frac{1}{3}} = (4^3)^{\frac{1}{3}} = 4^{3\times\frac{1}{3}} = 4^1 = 4$

(c) $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{3\times\frac{1}{3}} = 5^1 = 5$

(d) $243^{\frac{1}{5}} = (3^5)^{\frac{1}{5}} = 3^{5\times\frac{1}{5}} = 3^1 = 3$

(e) $729^{\frac{1}{6}} = (3^6)^{\frac{1}{6}} = 3^{6\times\frac{1}{6}} = 3^1 = 3$

(f) $1296^{\frac{1}{4}} = (6^4)^{\frac{1}{4}} = 6^{4\times\frac{1}{4}} = 6^1 = 6$

E.g. 2 Without using a calculator, find the value of:

(a) $27^{-\frac{1}{3}}$ (b) $256^{-\frac{1}{4}}$ (c) $\left(\frac{81}{100}\right)^{-\frac{1}{2}}$

(d) $\left(\frac{64}{216}\right)^{-\frac{1}{3}}$ (e) $\left(\frac{81}{625}\right)^{-\frac{1}{4}}$ (f) $\left(\frac{216}{27}\right)^{-\frac{1}{3}}$

Working: (a) $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{(3^3)^{\frac{1}{3}}} = \frac{1}{3^{3\times\frac{1}{3}}} = \frac{1}{3^1} = \frac{1}{3}$

or $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

(b) $256^{-\frac{1}{4}} = \frac{1}{256^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{256}} = \frac{1}{4}$

(c) $\left(\frac{81}{100}\right)^{-\frac{1}{2}} = \left(\frac{100}{81}\right)^{\frac{1}{2}} = \left(\left(\frac{10}{9}\right)^2\right)^{\frac{1}{2}} = \left(\frac{10}{9}\right)^{2\times\frac{1}{2}} = \frac{10}{9}$

or $\left(\frac{81}{100}\right)^{-\frac{1}{2}} = \left(\frac{100}{81}\right)^{\frac{1}{2}} = \sqrt{\frac{100}{81}} = \frac{10}{9}$

(d) $\left(\frac{64}{216}\right)^{-\frac{1}{3}} = \left(\frac{216}{64}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{216}{64}} = \sqrt[3]{\frac{6^3}{4^3}} = \frac{6}{4} = \frac{3}{2}$

(e) $\left(\frac{81}{625}\right)^{-\frac{1}{4}} = \left(\frac{625}{81}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{625}{81}} = \sqrt[4]{\frac{5^4}{3^4}} = \frac{5}{3}$

(f) $\left(\frac{216}{27}\right)^{-\frac{1}{3}} = \left(\frac{27}{216}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{27}{216}} = \sqrt[3]{\frac{3^3}{6^3}} = \frac{3}{6} = \frac{1}{2}$

E.g. 3 Simplify these expressions into the form x^n where n is an integer:

(a) $\sqrt[7]{x^{14}}$ (b) $\sqrt[4]{x^5 \times x^7}$ (c) $\sqrt{\frac{x^3}{x^{15}}}$ (d) $\frac{1}{\sqrt[3]{x^4 \times x^{14}}}$

Working: (a) $\sqrt[7]{x^{14}} = (x^{14})^{\frac{1}{7}} = x^{14\times\frac{1}{7}} = x^2$

(b) $\sqrt[4]{x^5 \times x^7} = \sqrt[4]{x^{5+7}} = \sqrt[4]{x^{12}} = (x^{12})^{\frac{1}{4}} = x^{12\times\frac{1}{4}} = x^3$

(c) $\sqrt{\frac{x^3}{x^{15}}} = \sqrt{x^{3-15}} = \sqrt{x^{-12}} = (x^{-12})^{\frac{1}{2}} = x^{-12\times\frac{1}{2}} = x^{-6}$

(d) $\frac{1}{\sqrt[3]{x^4 \times x^{14}}} = \frac{1}{\sqrt[3]{x^{18}}} = \frac{1}{(x^{18})^{\frac{1}{3}}} = \frac{1}{x^{18\times\frac{1}{3}}} = \frac{1}{x^6} = x^{-6}$

E.g. 4 Simplify: (a) $\sqrt{16x^{10}}$ (b) $\sqrt[3]{27y^{12}}$

Working: (a) $\sqrt{16x^{10}} = \sqrt{16} \times (x^{10})^{\frac{1}{2}} = 4x^{10\times\frac{1}{2}} = 4x^5$

(b) $\sqrt[3]{27y^{12}} = \sqrt[3]{27} \times (y^{12})^{\frac{1}{3}} = 3y^{12\times\frac{1}{3}} = 3y^4$

Video: Laws of indices
Video: Negative indices
Video: Fractional indices

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p45 E2.1 Qu 2, 3, 4, 5ace…, (numbers), 6ace… (algebra)
A*-G class textbook:	p41 E2.2 Qu 2, 3, 4, 5ace…, (numbers), 6ace… (algebra)
9-1 homework book:	p14 E2.1 Qu 1-5
A*-G homework book:	p11 E2.2 Qu 1-4

Homework book answers (only available during a lockdown)