Fractional Indices when the Numerator is not 1

Starter

1. **(Review of last lesson)**

Without a calculator, evaluate: (a) $625^{-\frac{1}{4}}$ (b) $\left(\frac{243}{32}\right)^{\frac{1}{5}}$

Working: (a) $625^{-\frac{1}{4}} = \frac{1}{625^{\frac{1}{4}}} = \frac{1}{(5^4)^{\frac{1}{4}}} = \frac{1}{5^{4\times\frac{1}{4}}} = \frac{1}{5^1} = \frac{1}{5}$

(b) $\left(\frac{243}{32}\right)^{\frac{1}{5}} = \left(\frac{3^5}{2^5}\right)^{\frac{1}{5}} = \frac{3}{2}$

2. **(Review of last lesson)** Simplify: (a) $\sqrt[3]{8a^{12}}$ (b) $\sqrt[4]{81t^{20}}$

Working: (a) $\sqrt[3]{8a^{12}} = \sqrt[3]{8} \times (a^{12})^{\frac{1}{3}} = 2a^{12\times\frac{1}{3}} = 2a^4$

(b) $\sqrt[4]{81t^{20}} = \sqrt[4]{81} \times (t^{20})^{\frac{1}{4}} = 3t^{20\times\frac{1}{4}} = 3t^5$

E.g. 2 Without a calculator, find the value of:

(a) $36^{\frac{3}{2}}$ (b) $27^{\frac{2}{3}}$ (c) $16^{\frac{5}{4}}$ (d) $\left(\frac{25}{49}\right)^{\frac{3}{2}}$

Working: (a) $36^{\frac{3}{2}} = \left(\sqrt{36}\right)^3 = 6^3 = 216$ ***do the root first***

(b) $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$ ***do the root first***

(c) $16^{\frac{5}{4}} = \left(\sqrt[4]{16}\right)^5 = 2^5 = 32$ ***do the root first***

(d) $\left(\frac{25}{49}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{25}{49}}\right)^3 = \left(\frac{5}{7}\right)^3 = \frac{125}{343}$ ***do the root first***

With ***negative indices***, the first step is to write the reciprocal of the number and change the sign of the power.

E.g. 3 Without a calculator, evaluate:

(a) $25^{-\frac{3}{2}}$ (b) $32^{-\frac{4}{5}}$ (c) $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

N.B. Continue to do the root first.

Working: (a) $25^{-\frac{3}{2}} = \left(\frac{1}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{1}{25}}\right)^{3} = \left(\frac{1}{5}\right)^{3} = \frac{1}{125}$

(b) $32^{-\frac{4}{5}} = \left(\frac{1}{32}\right)^{\frac{4}{5}} = \left(\sqrt[5]{\frac{1}{32}}\right)^{4} = \left(\frac{1}{2}\right)^{4} = \frac{1}{16}$

(c) $\left(\frac{27}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{8}{27}}\right)^{2} = \left(\frac{2}{3}\right)^{2} = \frac{4}{9}$

N.B. When a ***fraction*** is raised to a negative power, "flip" the fraction and make the power positive.

E.g. 4 Simplify: (a) $(8a^9)^{\frac{2}{3}}$ (b) $(27p^6q^3)^{\frac{4}{3}}$

(c) $(32a^5b^{10})^{\frac{4}{5}}$ (d) $\left(\frac{32a^{10}}{b^3}\right)^{-\frac{2}{5}}$

Hint: Separate the coefficients from the letters part as they need to be treated differently.

Working: (a) $(8a^9)^{\frac{2}{3}} = (\sqrt[3]{8})^2 \times a^{9\times\frac{2}{3}} = 2^2 \times a^6 = 4a^6$

(b) $(27p^6q^3)^{\frac{4}{3}} = (\sqrt[3]{27})^4 \times p^{6\times\frac{4}{3}} \times q^{3\times\frac{4}{3}} = 3^4 \times p^8q^4 = 81p^8q^4$

(c) $(32a^5b^{10})^{\frac{4}{5}} = (\sqrt[5]{32})^4 \times a^{5\times\frac{4}{5}} \times b^{10\times\frac{4}{5}} = 2^4 \times a^4b^8 = 16a^4b^8$

(d) $\left(\frac{32a^{10}}{b^3}\right)^{-\frac{2}{5}} = \left(\frac{b^3}{32a^{10}}\right)^{\frac{2}{5}} = \frac{b^{3\times\frac{2}{5}}}{\left(\sqrt[5]{32}\right)^2 \times a^{10\times\frac{2}{5}}} = \frac{b^{\frac{6}{5}}}{2^2 \times a^4} = \frac{b^{\frac{6}{5}}}{4a^4}$

Video: Laws of indices
Video: Negative indices
Video: Fractional indices

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p47 E2.2 Qu 1, 2ace…, 3-5, 6ace…
A*-G class textbook:	p43 E2.3 Qu 1, 2ace…, 3-6, 7ace…
9-1 homework book:	p14 E2.2 Qu 1-6
A*-G homework book:	p12 E2.3 Qu 1-5

Homework book answers (only available during a lockdown)