

Indices (Mixed Problems)

Starter

1. **(Review of last lesson)** Estimate $\sqrt{137}$.

Working: The nearest square numbers above and below 137 are 121 and 144.
 $\sqrt{121} = 11$ and $\sqrt{144} = 12$
 $\sqrt{121} < \sqrt{137} < \sqrt{144}$
 $11 < \sqrt{137} < 12$
 137 is closer to 144 than 121 so $\sqrt{137}$ will be closer to 12 than 11
 $\sqrt{137} \approx 11.7$ *(between 11.6 and 11.8 would be fine)*

(Review of Indices material)

2. Simplify: a) $\frac{(a^3)^6 \times a^7}{a^{10}}$ b) $(2p^3q^7)^4$

Working: (a) $\frac{(a^3)^6 \times a^7}{a^{10}} = \frac{a^{18} \times a^7}{a^{10}} = \frac{a^{25}}{a^{10}} = a^{15}$

(b) $(2p^3q^7)^4 = 2^4 \times p^{3 \times 4} \times q^{7 \times 4} = 16p^{12}a^{28}$

3. Evaluate (a) $\left(\frac{7}{8}\right)^{-1}$ (b) $\left(\frac{6}{5}\right)^{-2}$ (c) $\left(\frac{1}{7}\right)^{-2}$
 (d) $64^{\frac{1}{3}}$ (e) $16^{-\frac{1}{4}}$ (f) $\left(\frac{81}{100}\right)^{-\frac{1}{2}}$
 (g) $27^{\frac{2}{3}}$ (h) $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

Working: (a) $\left(\frac{7}{8}\right)^{-1} = \frac{8}{7}$ *invert the fraction*

(b) $\left(\frac{6}{5}\right)^{-2} = \left(\frac{5}{6}\right)^2 = \frac{5^2}{6^2} = \frac{25}{36}$

(c) $\left(\frac{1}{7}\right)^{-2} = \left(\frac{7}{1}\right)^2 = 7^2 = 49$

(d) $64^{\frac{1}{3}} = (4^3)^{\frac{1}{3}} = 4^1 = 4$

(e) $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{(2^4)^{\frac{1}{4}}} = \frac{1}{2^1} = \frac{1}{2}$

(f) $\left(\frac{81}{100}\right)^{-\frac{1}{2}} = \left(\frac{100}{81}\right)^{\frac{1}{2}} = \sqrt{\frac{100}{81}} = \frac{10}{9}$

(g) $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

(h) $\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

3. Simplify: (a) $(8a^9)^{\frac{2}{3}}$ (b) $(32a^{10})^{-\frac{2}{5}}$

Working: (a) $(8a^9)^{\frac{2}{3}} = 8^{\frac{2}{3}} \times (a^9)^{\frac{2}{3}} = (\sqrt[3]{8})^2 \times a^{9 \times \frac{2}{3}} = 2^2 \times a^6 = 4a^6$

(b) $(32a^{10})^{-\frac{2}{5}} = \frac{1}{(32a^{10})^{\frac{2}{5}}} = \frac{1}{32^{\frac{2}{5}} \times (a^{10})^{\frac{2}{5}}} = \frac{1}{(2^5)^{\frac{2}{5}} \times a^{10 \times \frac{2}{5}}} = \frac{1}{2^2 \times a^4} = \frac{1}{4a^4}$

4. Solve: (a) $3^{2x+7} = 81$ (b) $2^{3-5x} = \frac{1}{32}$ (c) $9^{8x-5} = \left(\frac{1}{27}\right)^2$

Working: (a) **Replace 81 by 3^4 :** $3^{2x+7} = 3^4$
Equating powers of 3: $2x + 7 = 4$
 $2x = -3$
 $x = -\frac{3}{2}$

(b) **Replace 32 by 2^5 :** $2^{3-5x} = \frac{1}{2^5}$
 $2^{3-5x} = 2^{-5}$
Equating powers of 2: $3 - 5x = -5$
 $-5x = -8$
 $x = \frac{8}{5}$

(c) **The number 3 connects 9 and 27.**
Replace 9 by 3^2 and 27 by 3^3 : $(3^2)^{8x-5} = \left(\frac{1}{3^3}\right)^2$
3rd law of logs: $(3^2)^{8x-5} = \frac{1}{3^6}$
 $3^{16x-10} = 3^{-6}$
3rd law of logs: $16x - 10 = -6$
Equating powers of 3: $16x = 4$
 $x = \frac{1}{4}$

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:

p52 E2.5 Qu 1ace..., 3, 5, 6, 8ace..., 9, 12*, 13*

A*-G class textbook:

p46 E2.5 Qu 1ace..., 3, 5, 6, 7ace..., 8, 11*, 12*

9-1 homework book:

p18 E2.5 Qu 1ace, 2aceg, 3-7, 8ace, 9, 10

A*-G homework book:

p13 E2.5 Qu 1ace, 2aceg, 3-6, 7ace, 8

Homework book answers (only available during a lockdown)