

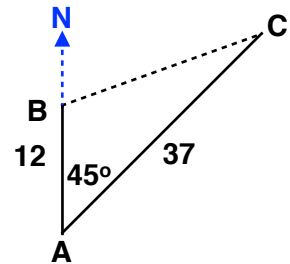
Mixed Sine and Cosine Rule

Starter

1. **(Review of last lesson)** Village B is 12 miles north of village A. Village C is 37 miles north east of village A. Find the direct distance between village B and village C, correct to 3 s.f.

Working: Draw a diagram.
North East \equiv bearing of 045°

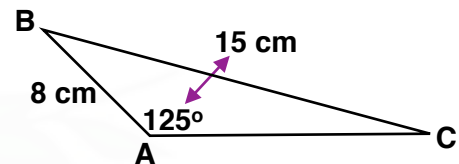
Finding side: $a^2 = b^2 + c^2 - 2bc \cos A$
 $BC^2 = 12^2 + 37^2 - 2 \times 12 \times 37 \cos 45$
 $BC = 29.8$
 The distance between B and C is 29.8 miles (3 s.f.)



- E.g. 1** In a triangle ABC, $\angle BAC = 125^\circ$, $BC = 15$ cm and $AB = 8$ cm. Find $\angle ABC$.

Working: Draw a diagram.
Arrow between knowns \Rightarrow sine rule
Cannot find $\angle ABC$ directly
Need to find $\angle ACB$ first

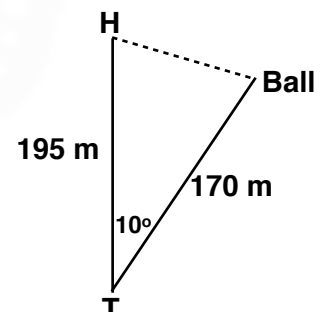
Finding an angle: $\frac{\sin A}{a} = \frac{\sin B}{b}$
 $\frac{\sin C}{8} = \frac{\sin 125}{15} \Rightarrow \sin C = \frac{8 \sin 125}{15}$
 $C = \sin^{-1}\left(\frac{8 \sin 125}{15}\right) = 22.47^\circ$
 $B = 180 - 125 - 22.47 = 29.1^\circ$ (3 s.f.)



- E.g. 2** A golfer hits her ball a distance of 170 m on a hole which measures 195 m from the tee, T, to the green, G. If her shot is directed 10° away from the direct line to the green, find the distance between her ball and the green.

Working: Draw a diagram.
No arrow between knowns \Rightarrow cosine rule

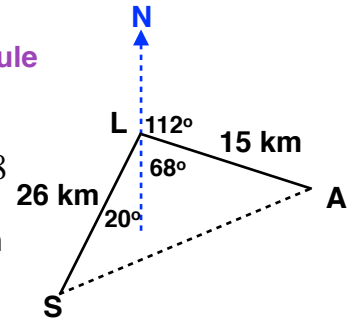
Finding side: $a^2 = b^2 + c^2 - 2bc \cos A$
 $BH^2 = 195^2 + 170^2 - 2 \times 195 \times 170 \cos 10$
 $BH = 40.4$
 The distance between the ball and the hole is 40.4 m (3 s.f.)



- E.g. 3** From a lighthouse, L, an aircraft carrier, A, is 15 km away on a bearing of 112° and a submarine, S, is 26 km away on a bearing of 200° . Find:
- the distance between A and S to 3 s.f..
 - the bearing of A from S to 1 d.p..

Working: (a) **Draw a diagram.**
No arrow between knowns \Rightarrow cosine rule

Finding side: $a^2 = b^2 + c^2 - 2bc \cos A$
 $AS^2 = 26^2 + 15^2 - 2 \times 26 \times 15 \cos 88^\circ$
 $AS = 29.6$
 The distance between A and S is 29.6 km (3 s.f.)



(b) **Bearing of A from S – start from S**
Draw a North line up from S.

By alternate angles, the angle between the North line and LS is 20° .
 The required bearing is $20^\circ + \angle LSA$

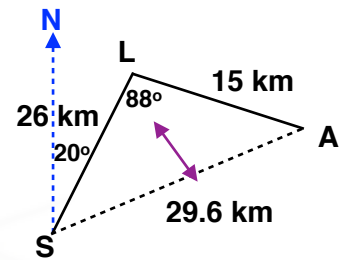
Arrow between knowns \Rightarrow sine rule

$$\text{Finding an angle: } \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin S}{15} = \frac{\sin 88^\circ}{29.6} \Rightarrow \sin S = \frac{15 \sin 88^\circ}{29.6}$$

$$S = \sin^{-1}\left(\frac{15 \sin 88^\circ}{29.6}\right) = 30.42^\circ$$

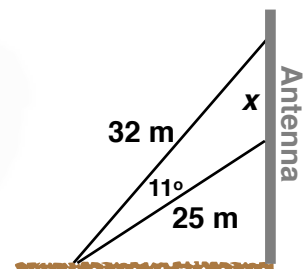
The bearing of A from S is 050.4° (1 d.p.)



- E.g. 4** Starting from the same point on the ground two wires of length 25 m and 32 m are attached to different points on a vertical antenna. The angle between the wires is 11° . Find the distance between where the wires are attached to the antenna.

Working: **Draw a diagram.**
No arrow between knowns \Rightarrow cosine rule

Finding side: $a^2 = b^2 + c^2 - 2bc \cos A$
 $x^2 = 25^2 + 32^2 - 2 \times 25 \times 32 \cos 11^\circ$
 $x = 8.85$
 The distance between where the wires are attached to the antenna is 8.85 m (3 s.f.)



Solutions to Starter and E.g.s

Exercise

- 9-1 class textbook: p583 E18.3 Qu 1, 2, 3xyzxyx, 4-8; p584 E18.4 Qu 1-8
 A*-G class textbook: p543 Ex 18.3 Qu 1, 2, 3xyzxyx, 4-11; p555-557 Qu 4, 8, 9
 9-1 homework book: p197 E18.3 Qu 1-6, p198 E18.4 Qu 1-8
 A*-G homework book: p151 E18.3 Qu 1-7

Homework book answers (only available during a lockdown)