Problems Leading to Quadratics

Starter

1. **(Review of last lesson)** Solve the equation $5x + 12 = (x + 1)(x + 7)$, giving your answers to 3 s.f..

Working: Expand: $5x + 12 = x^2 + 8x + 7 \quad \Rightarrow \quad x^2 + 3x - 5 = 0$

$a = 1 \qquad b = 3 \qquad c = -5$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-3 \pm \sqrt{3^2 - 4 \times 1 \times (-5)}}{2 \times 1}$

$x = \dfrac{-3 \pm \sqrt{9 - -20}}{2}$

$x = \dfrac{-3 + \sqrt{29}}{2}$ or $x = \dfrac{-3 - \sqrt{29}}{2}$

$x = 1.19$ or $x = -4.19$ (3 s.f.)

2. Solve the equation $2x + 5 + \dfrac{1}{x} = 0$, giving your answers exactly

Working: Multiply by x to make the equation a quadratic: $2x^2 + 5x + 1 = 0$

$a = 2 \qquad b = 5 \qquad c = 1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-5 \pm \sqrt{5^2 - 4 \times 2 \times 1}}{2 \times 2}$

$x = \dfrac{-5 \pm \sqrt{25 - 8}}{4}$

$x = \dfrac{-5 + \sqrt{17}}{4}$ or $x = \dfrac{-5 - \sqrt{17}}{4}$

E.g. 1 The square of a number plus the original number is 22. Give your answers to 3 s.f..

Working: The square of a number… $\Rightarrow \quad x^2$

The square of a number plus the original number… $\Rightarrow \quad x^2 + x$

The square of a number plus the original number is 22. $\Rightarrow \quad x^2 + x = 22$

$x^2 + x = 22 \quad \Rightarrow \quad x^2 + x - 22 = 0$

$a = 1 \qquad b = 1 \qquad c = -22$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-22)}}{2 \times 1}$

$x = \dfrac{-1 \pm \sqrt{1 - -88}}{2}$

$x = \dfrac{-1 + \sqrt{89}}{2}$ or $x = \dfrac{-1 - \sqrt{89}}{2}$

$x = 4.22$ or $x = -5.22$ (3 s.f.)

E.g. 2 The sides of a right-angle triangle are x, $x+7$ and $x+12$. Find the value of x to 3 s.f..

Working:

By Pythagoras: $x^2 + (x+7)^2 = (x+12)^2$

Expand: $x^2 + x^2 + 14x + 49 = x^2 + 24x + 144$

$x^2 - 10x - 95 = 0$

$a = 1 \qquad b = -10 \qquad c = -95$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 1 \times (-95)}}{2 \times 1}$

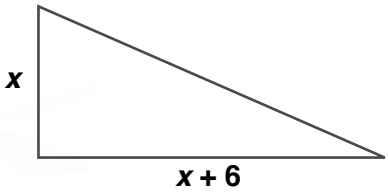
$x = \dfrac{10 \pm \sqrt{100 - -380}}{2}$

$x = \dfrac{10 + \sqrt{480}}{2}$ or $\quad x = \dfrac{10 - \sqrt{480}}{2}$

$x = 15.95$ or $\quad x = -5.95$ (3 s.f.)

Since $x > 0$, $x = 16.0$

E.g. 3 The area of this right-angled triangle is 17 m². Find the value of x to 3 s.f..



Working:

Area of triangle $= \dfrac{\text{base} \times \text{height}}{2}$

$\dfrac{x(x+6)}{2} = 17$

$x^2 + 6x = 34 \quad \Rightarrow \quad x^2 + 6x - 34 = 0$

$a = 1 \qquad b = 6 \qquad c = -34$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times (-34)}}{2 \times 1}$

$x = \dfrac{-6 \pm \sqrt{36 - -136}}{2}$

$x = \dfrac{-6 + \sqrt{172}}{2}$ or $\quad x = \dfrac{-6 + \sqrt{172}}{2}$

$x = 3.56$ or $\quad x = -9.56$ (3 s.f.)

Since $x > 0$, $x = 3.56$

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p402 E12.7 Qu 1-10
A*-G class textbook:	p361 E12.4 Qu 1-10
9-1 homework book:	p402 E12.7 Qu 1-6
A*-G homework book:	p102 E12.4 Qu 1-6

Homework book answers (only available during a lockdown)