

Rationalise the Denominator

Starter

1. **(Review of Y11 material)** Without a calculator, evaluate $\left(\frac{25}{49}\right)^{\frac{3}{2}}$

Working: $\left(\frac{25}{49}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{25}{49}}\right)^3 = \left(\frac{5}{7}\right)^3 = \frac{125}{343}$

2. **(Review of Y11 material)** Solve $25^{4x+1} = 125^2$.

Working: *The number 5 connects 25 and 125.*

Replace 25 by 5^2 and 125 by 5^3 : $(5^2)^{4x+1} = (5^3)^2$
 $5^{8x+2} = 5^6$

3rd law of logs:

Equating powers of 3:

$$8x + 2 = 6$$

$$8x = 4$$

$$x = \frac{1}{2}$$

3. Simplify: (a) $\sqrt{8}$ (b) $\sqrt{45}$

Working: (a) $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

(b) $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

4. Expand $(3 - \sqrt{2})(3 + \sqrt{2})$.

Working: $(3 - \sqrt{2})(3 + \sqrt{2}) = 9 + 3\sqrt{2} - 3\sqrt{2} - 2 = 7$

E.g. 1 Rationalise $\frac{5}{\sqrt{17}}$.

N.B. If $\sqrt{17}$ is multiplied by $\sqrt{17}$, it becomes 17.

Working: $\frac{5}{\sqrt{17}} = \frac{5}{\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}}$
 $= \frac{5\sqrt{17}}{17}$

multiplying by $\frac{\sqrt{17}}{\sqrt{17}}$ is like multiplying by 1

E.g. 2 Rationalise: (a) $\frac{6}{\sqrt{10}}$ (b) $\frac{8}{3\sqrt{18}}$

Working: (a) $\frac{6}{\sqrt{10}} = \frac{6}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{6\sqrt{10}}{10} = \frac{3\sqrt{10}}{5}$

(b) $\frac{8}{3\sqrt{18}} = \frac{8}{3\sqrt{18}} \times \frac{\sqrt{18}}{\sqrt{18}} = \frac{8\sqrt{18}}{3 \times 18} = \frac{8 \times 3\sqrt{2}}{3 \times 18} = \frac{8\sqrt{2}}{18} = \frac{4\sqrt{2}}{9}$

E.g. 3 When $3 + \sqrt{2}$ is multiplied by $3 - \sqrt{2}$ the surd is eliminated.
 Multiply the following by a suitable bracket to eliminate the surds.
 Expand the two brackets to find the integer.

(a) $3 + \sqrt{5}$ (b) $\sqrt{6} - 2$ (c) $\sqrt{14} - \sqrt{7}$ (d) $a + \sqrt{b}$

Working: (a) Multiply by $3 - \sqrt{5}$
 $(3 + \sqrt{5})(3 - \sqrt{5}) = 3^2 - 3\sqrt{5} + 3\sqrt{5} - (\sqrt{5})^2 = 9 - 5 = 4$

(b) Multiply by $\sqrt{6} + 2$
 $(\sqrt{6} - 2)(\sqrt{6} + 2) = (\sqrt{6})^2 - 2^2 = 6 - 4 = 2$

(c) Multiply by $\sqrt{14} + \sqrt{7}$
 $(\sqrt{14} - \sqrt{7})(\sqrt{14} + \sqrt{7}) = (\sqrt{14})^2 - (\sqrt{7})^2 = 14 - 7 = 7$

(d) Multiply by $a - \sqrt{b}$
 $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - (\sqrt{b})^2 = a^2 - b$

E.g. 4 Rationalise: (a) $\frac{1}{\sqrt{2} - 1}$ (b) $\frac{6}{8 + \sqrt{5}}$ (c) $\frac{3}{\sqrt{13} - 2}$

Working: (a) $\frac{1}{\sqrt{2} - 1} = \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{\sqrt{2} + 1}{2 - 1} = \sqrt{2} + 1$

(b) $\frac{6}{8 + \sqrt{5}} = \frac{6}{8 + \sqrt{5}} \times \frac{8 - \sqrt{5}}{8 - \sqrt{5}} = \frac{6(8 - \sqrt{5})}{64 - 5} = \frac{48 - 6\sqrt{5}}{59}$

(c) $\frac{3}{\sqrt{13} - 2} = \frac{3}{\sqrt{13} - 2} \times \frac{\sqrt{13} + 2}{\sqrt{13} + 2} = \frac{3(\sqrt{13} + 2)}{13 - 4} = \frac{\sqrt{13} + 2}{3}$

E.g. 5 Rationalise: (a) $\frac{3}{\sqrt{3} - \sqrt{2}}$ (b) $\frac{4\sqrt{3}}{2\sqrt{3} - 3}$ (c) $\frac{1}{3\sqrt{2} - 2\sqrt{3}}$

Working: (a)
$$\begin{aligned}\frac{3}{\sqrt{3} - \sqrt{2}} &= \frac{3}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \\ &= \frac{3(\sqrt{3} + \sqrt{2})}{3 - 2} \\ &= 3(\sqrt{3} + \sqrt{2})\end{aligned}$$

(b)
$$\begin{aligned}\frac{4\sqrt{3}}{2\sqrt{3} - 3} &= \frac{4\sqrt{3}}{2\sqrt{3} - 3} \times \frac{2\sqrt{3} + 3}{2\sqrt{3} + 3} \\ &= \frac{4\sqrt{3}(2\sqrt{3} + 3)}{12 - 9} \\ &= \frac{4\sqrt{3}(2\sqrt{3} + 3)}{3} \\ &= \frac{24 + 12\sqrt{3}}{3} \\ &= 8 + 4\sqrt{3} \\ &= 4(2 + \sqrt{3})\end{aligned}$$

(c)
$$\begin{aligned}\frac{1}{3\sqrt{2} - 2\sqrt{3}} &= \frac{1}{3\sqrt{2} - 2\sqrt{3}} \times \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} \\ &= \frac{3\sqrt{2} + 2\sqrt{3}}{18 - 12} \\ &= \frac{3\sqrt{2} + 2\sqrt{3}}{6}\end{aligned}$$

E.g. 6 Rationalise: (a) $\frac{3 - \sqrt{5}}{\sqrt{5} + 1}$ (b) $\frac{2 + \sqrt{7}}{6 - \sqrt{7}}$

Working: (a)
$$\begin{aligned}\frac{3 - \sqrt{5}}{\sqrt{5} + 1} &= \frac{3 - \sqrt{5}}{\sqrt{5} + 1} \times \frac{\sqrt{5} - 1}{\sqrt{5} - 1} \\ &= \frac{3\sqrt{5} - 3 - 5 + \sqrt{5}}{5 - 1} \\ &= \frac{4\sqrt{5} - 8}{4} \\ &= \sqrt{5} - 2\end{aligned}$$

(b)
$$\begin{aligned}\frac{2 + \sqrt{7}}{6 - \sqrt{7}} &= \frac{2 + \sqrt{7}}{6 - \sqrt{7}} \times \frac{6 + \sqrt{7}}{6 + \sqrt{7}} \\ &= \frac{12 + 2\sqrt{7} + 6\sqrt{7} + 7}{36 - 7} \\ &= \frac{19 + 8\sqrt{7}}{29}\end{aligned}$$

Video: [Rationalising denominators](#)

Video: [Surds \(addition\)](#)

Video: [Surds \(expanding brackets\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

There are very questions in the textbook like the ones above.

9-1 class textbook: p16 E1.4 Qu 5acegi, 6–9

A*-G class textbook: p15 E1.3 Qu 7, 8acegi, 9–10

9-1 homework book: p6 E1.4 Qu 3acegil, 4–6

A*-G homework book: p4 E1.3 Qu 6acegil

[Homework book answers \(only available during a lockdown\)](#)