

Simultaneous Equations (Linear/Non-Linear)

Starter

1. **(Review of last lesson)** The sides of a right-angle triangle are x , $x + 1$ and $3x$. Find the value(s) of x to 3 s.f..

Working: By Pythagoras: $x^2 + (x + 1)^2 = (3x)^2$
 Expand: $x^2 + x^2 + 2x + 1 = 9x^2$
 $7x^2 - 2x - 1 = 0$
 $a = 7$ $b = -2$ $c = -1$
 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 7 \times (-1)}}{2 \times 7}$
 $x = \frac{2 \pm \sqrt{4 + 28}}{14}$
 Since $x > 0$, $x = \frac{2 + \sqrt{32}}{14}$
 $x = 0.547$ (3 s.f.)

2. **(Review of Y9 material)** Expand $(x + 5)(x - 4)$.

Working: $(x + 5)(x - 4) = x^2 - 4x + 5x - 20 = x^2 + x - 20$

3. **(Review of Y10 material)** Solve $x^2 + 3x - 10 = 0$ by factorising.

Working: $1 \times (-10) = -10$ *coefficient of x^2 \times constant term*
Looking for 2 numbers that multiply to give -10 and add to give 3
 Multiply: $-10 = 5 \times (-2)$
 Add: $3 = 5 + (-2)$
 $x^2 + 5x - 2x - 10 = 0$ *split the $3x$ term into $5x - 2x$*
 $x(x + 5) - 2(x + 5) = 0$ *factorise by grouping*
 $(x + 5)(x - 2) = 0$
 $x = -5$ or $x = 2$

- E.g. 2** Solve the simultaneous equations $y = x^2 - 2x$ and $y = x + 4$ to find the coordinates where the curve meets the line.

Working: Linear function is $y = x + 4$ – there is no need to rearrange.
 Substitute into $y = x^2 - 2x$: $x + 4 = x^2 - 2x$
 $x^2 - 3x - 4 = 0$
 Factorise (or formula) to solve: Multiply: $-4 = -4 \times 1$
 Add: $-3 = -4 + 1$
 $x^2 - 4x + x - 4 = 0$
 $x(x - 4) + 1(x - 4) = 0$
 $(x - 4)(x + 1) = 0$
 $x = 4$ or $x = -1$
 When $x = 4$, $y = 4 + 4 = 8$
 When $x = -1$, $y = -1 + 4 = 3$
 The curve and line meet at the points $(4, 8)$ and $(-1, 3)$

E.g. 3 Solve the equations $x + y = 5$ and $2x^2 + xy = 14$ simultaneously.

Working: $x + y = 5$ is the linear function
Rearrange to make y the subject: $y = 5 - x$
Substitute into $2x^2 + xy = 14$: $2x^2 + x(5 - x) = 14$
 $2x^2 + 5x - x^2 = 14$
 $x^2 + 5x - 14 = 0$
Factorise (or formula) to solve: Multiply: $-14 = 7 \times (-2)$
Add: $5 = 7 + (-2)$
 $x^2 + 7x - 2x - 14 = 0$
 $x(x + 7) - 2(x + 7) = 0$
 $(x + 7)(x - 2) = 0$
 $x = -7$ or $x = 2$
When $x = -7$, $y = 5 - -7 = 12$
When $x = 2$, $y = 5 - 2 = 3$
The curve and line meet at the points $(-7, 12)$ and $(2, 3)$

Video: [Simultaneous equations \(linear/non-linear\)](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p404 E12.8 Qu 7-16
A*-G class textbook: p364 E12.5 Qu 7-16
9-1 homework book: p138 E12.8 Qu 1-6
A*-G homework book: p103 E12.5 Qu 1-5

[Homework book answers \(only available during a lockdown\)](#)