Solving Quadratics with the Formula

Starter

1. **(Review of last lesson)** Write down the turning point of the parabola $y = (x-4)^2 - 13$.

Working: For $x-$coordinate, put the expression in the bracket equal to zero and solve:

$$x - 4 = 0 \quad \Rightarrow \quad x = 4$$

For $y-$coordinate, put the bracket equal to zero and solve:

$$y = 0^2 - 13 = -13$$

The coordinates of the turning point are $(4, -13)$

2. **(Review of last lesson)** Find the coordinates of the turning point of the parabolas:

(a) $y = x^2 + 14x + 54$. (b) $3x^2 - 36x + 2$

Working: (a) $y = x^2 + 14x + 54 \equiv (x+7)^2 - 7^2 + 54 \equiv (x+7)^2 + 5$

The coordinates of the turning point are $(-7, 5)$

See question 1 above for full working.

(b) $$\begin{aligned} 3x^2 - 36x + 2 &\equiv 3\left[x^2 - 12x\right] + 2 \\ &\equiv 3\left[(x-6)^2 - 6^2\right] + 2 \\ &\equiv 3\left[(x-6)^2 - 36\right] + 2 \\ &\equiv 3(x-6)^2 - 108 + 2 \\ &\equiv 3(x-6)^2 - 106 \end{aligned}$$

The coordinates of the turning point are $(6, -106)$

See question 1 above for full working.

E.g. Solve $7x^2 - 3x - 1 = 0$ using the quadratic formula.

Working: $a = 7 \qquad b = -3 \qquad c = -1$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 7 \times (-1)}}{2 \times 7}$

$$x = \frac{3 \pm \sqrt{9 + 28}}{14}$$

$$x = \frac{3 + \sqrt{37}}{14} \quad \text{or} \quad x = \frac{3 - \sqrt{37}}{14}$$

$$x = 0.649 \quad \text{or} \quad x = -0.220$$

E.g. 1 Solve $x^2 + x - 8 = 0$ giving your answer to 3 s.f.

Working: $a = 1 \qquad b = 1 \qquad c = -8$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $\quad x = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-8)}}{2 \times 1}$

$$x = \frac{1 \pm \sqrt{1 + 32}}{2}$$

$$x = \frac{1 + \sqrt{33}}{2} \quad \text{or} \quad x = \frac{1 - \sqrt{33}}{2}$$

$$x = 2.37 \quad \text{or} \quad x = -3.37$$

N.B. A question that involves solving a quadratic equation and includes the phrase "giving your answer to 3 s.f." means it will not factorise — so you should use the quadratic formula

E.g. 2 Solve these equation, giving your answer to 3 s.f.
(a) $x^2 - 2x - 5 = 0$ (b) $3x^2 + 7x - 5 = 0$

Working: (a) $a = 1 \quad b = -2 \quad c = -5$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-5)}}{2 \times 1}$

$x = \dfrac{2 \pm \sqrt{4 + 20}}{2}$

$x = \dfrac{2 + \sqrt{24}}{2}$ or $x = \dfrac{2 - \sqrt{24}}{2}$

$x = 3.45$ or $x = -1.45$

(b) $a = 3 \quad b = 7 \quad c = -5$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \dfrac{-7 \pm \sqrt{7^2 - 4 \times 3 \times (-5)}}{2 \times 3}$

$x = \dfrac{-7 \pm \sqrt{49 + 60}}{6}$

$x = \dfrac{-7 + \sqrt{109}}{6}$ or $x = \dfrac{-7 - \sqrt{109}}{6}$

$x = 0.573$ or $x = -2.91$

E.g. 3 Solve these equation, giving your answer to 3 s.f.
(a) $5x + x^2 - 4 = 0$ (b) $5 - 9x + 2x^2 = 0$
Hint: Rewrite the equation with terms in the correct order.

Working: (a) ***Rewrite the equation so that it is in the correct order.***

$5x + x^2 - 4 = 0 \quad \Rightarrow \quad x^2 + 5x - 4 = 0$

$a = 1 \quad b = 5 \quad c = -4$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \dfrac{-5 \pm \sqrt{5^2 - 4 \times 1 \times (-4)}}{2 \times 1}$

$x = \dfrac{-5 \pm \sqrt{25 + 16}}{2}$

$x = \dfrac{-5 + \sqrt{41}}{2}$ or $x = \dfrac{-5 - \sqrt{41}}{2}$

$x = 0.702$ or $x = -5.70$

(b) ***Rewrite the equation so that it is in the correct order.***

$5 - 9x + 2x^2 = 0 \quad \Rightarrow \quad 2x^2 - 9x + 5 = 0$

$a = 2 \quad b = -9 \quad c = 5$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times 5}}{2 \times 2}$

$$x = \frac{9 \pm \sqrt{81 - 40}}{4}$$

$$x = \frac{9 + \sqrt{41}}{4} \quad \text{or} \quad x = \frac{9 - \sqrt{41}}{4}$$

$$x = 3.85 \quad \text{or} \quad x = 0.649$$

Make sure the equation equals zero before using the formula — it is best to have the coefficient of x^2 equal to zero.

E.g. 4 Solve these equations, giving your answer exactly:
(a) $x(3x + 1) = 5$ (b) $x^2 - 5x + 11 = 2x + 3$
N.B. In this case an exact answer means leaving it in surd form.

Working: (a) Expand: $3x^2 + x = 5 \quad \Rightarrow \quad 3x^2 + x - 5 = 0$

$a = 3 \quad b = 1 \quad c = -5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-5)}}{2 \times 3}$

$$x = \frac{-1 \pm \sqrt{1 + 60}}{6}$$

$$x = \frac{-1 + \sqrt{61}}{6} \quad \text{or} \quad x = \frac{-1 - \sqrt{61}}{6}$$

(b) $x^2 - 5x + 11 = 2x + 3 \quad \Rightarrow \quad x^2 - 7x + 8 = 0$

$a = 1 \quad b = -7 \quad c = 8$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$: $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 8}}{2 \times 1}$

$$x = \frac{7 \pm \sqrt{49 - 32}}{2}$$

$$x = \frac{7 + \sqrt{17}}{2} \quad \text{or} \quad x = \frac{7 - \sqrt{17}}{2}$$

Video: Deriving the quadratic formula
Video: Quadratic formula

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p401 E12.6 Qu 1-18 odd (3 s.f.), 19-24 (p ± √q / r)
A*-G class textbook:	p360 E12.3 Qu 1-18 odd (3 s.f.), 19-24 (p ± √q / r)
9-1 homework book:	p137 E12.6 Qu 1-12
A*-G homework book:	p101 E12.3 Qu 1-12

Homework book answers (only available during a lockdown)