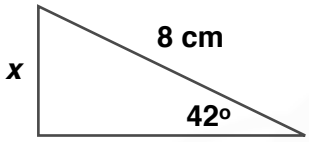
Trigonometry in 3-D

Starter

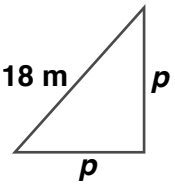
1. **(Review of Y9 material)** Find the missing side or angle in these right angle triangles:

N.B. We do not use the sine or cosine rule in right-angle triangles

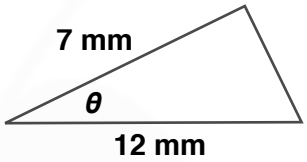
(a)



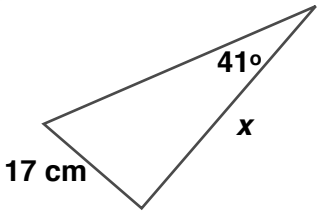
(b)



(c)

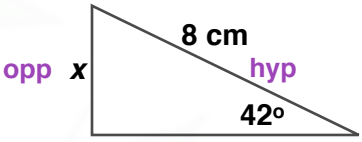


(d)



Working:

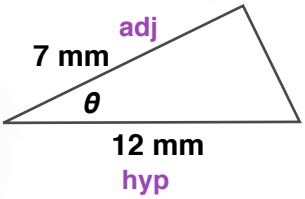
(a) **Label the given and unknown side**
Choose which trig. ratio to use: sine



$$\sin 42 = \frac{x}{8}$$
$$x = 8 \sin 42 = 5.35 \text{ cm}$$

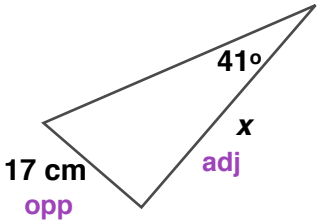
(b) By Pythagoras:
$$p^2 + p^2 = 18^2$$
$$2p^2 = 324$$
$$p^2 = 162 \quad \text{divide by 2}$$
$$p = \sqrt{162} = 12.7 \text{ m}$$

(c) **Label the given sides**
Choose which trig. ratio to use: sine



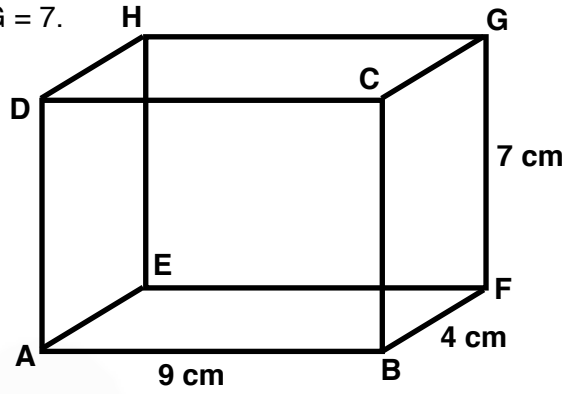
$$\cos\theta = \frac{7}{12}$$
$$\theta = \cos^{-1}\frac{7}{12} = 54.3°$$

(d) **Label the given and unknown side**
Choose which trig. ratio to use: tan



$$\tan 41 = \frac{17}{x}$$
$$x \tan 41 = 17$$
$$x = \frac{17}{\tan 41} = 19.6 \text{ cm}$$

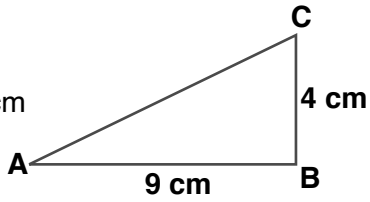
E.g. 1 In the cuboid ABCDEFGH, AB = 9, BF = 4, FG = 7.
Find the length of:
(a) AF
(b) AG



Working: (a) By Pythagoras: $4^2 + 9^2 = AF^2$

$AF^2 = 97$

$AF = 9.85$ cm



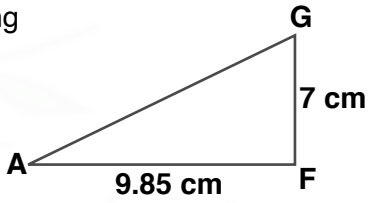
(b) By Pythagoras: $7^2 + AF^2 = AG^2$

Replace AF^2 by 97 rather than replacing AF by 9.85 to avoid rounding error.

$7^2 + 97 = AG^2$

$AG^2 = 146$

$AG = 12.1$ cm

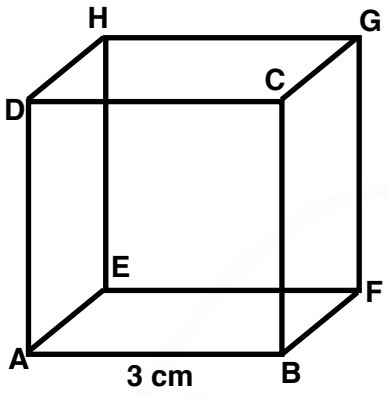


E.g. 2 A cube has sides of length 3 cm. Find:

(a) the exact value of AF

(b) the exact value of DF

(c) the angle DFA.



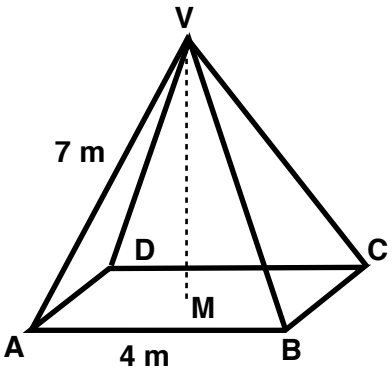
E.g. 3 A square-based pyramid has base ABCD where AB = 4 m. The point M is the midpoint of the square, which is vertically below the vertex, V.

Given that AV = 7 m, find:

(a) the exact length of AC

(b) the exact vertical height of the pyramid (i.e. MV)

(c) the $\angle AVB$

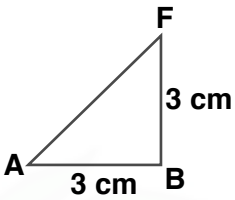


E.g. 2 ***Working:***

(a) By Pythagoras: $3^2 + 3^2 = AF^2$

$AF^2 = 18$

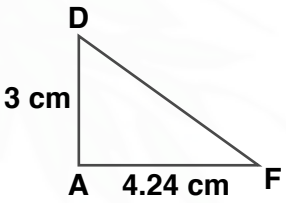
$AF = 3\sqrt{2} \approx 4.24$ cm



N.B. $\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$

(b) By Pythagoras: $3^2 + AF^2 = DF^2$

Replace AF^2 by 18 rather than replacing AF by 4.24 to avoid rounding error.



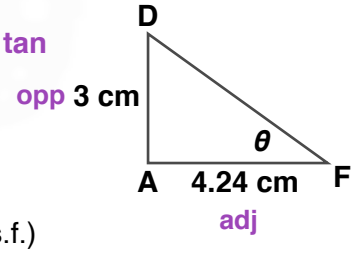
$3^2 + 18 = DF^2$

$DF^2 = 27$

$DF = 3\sqrt{3} \approx 5.20$ cm

(c) **Label the given sides**

Choose which trig. ratio to use: tan



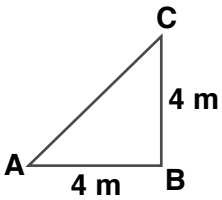
$$\tan\theta = \frac{3}{3\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{3}{3\sqrt{2}}\right) = 35.3^\circ \text{ (3 s.f.)}$$

E.g. 3 ***Working:*** (a) By Pythagoras: $4^2 + 4^2 = AC^2$

$AC^2 = 32$

$AF = \sqrt{32} = 4\sqrt{2} \approx 5.66$ m



N.B. $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

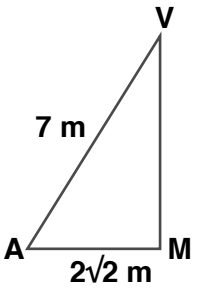
(b) $AM = \frac{1}{2}AC = \frac{1}{2} \times 4\sqrt{2} = 2\sqrt{2}$

By Pythagoras: $(2\sqrt{2})^2 + MV^2 = 7^2$

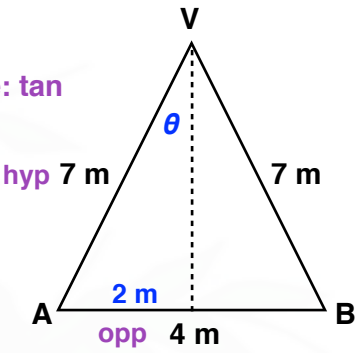
$8 + MV^2 = 49$

$MV = \sqrt{41} \approx 6.40$ m

The exact vertical height is $\sqrt{41}$ m



(c) **Split $\triangle AVB$ into 2 equal right-angled triangles.**
Label the given sides
Choose which trig. ratio to use: tan



$\tan\theta = \frac{2}{7}$

$\theta = \tan^{-1}\left(\frac{2}{7}\right) = 15.94°$

$\angle AVB = 2\tan^{-1}\left(\frac{2}{7}\right) = 31.9°$ (3 s.f.)

Distance between 2 points in 3-D

In 2-D, to find the distance between two points $(x_1,\ y_1)$ and $(x_2,\ y_2)$ we use Pythagoras. The formula is:

$$\text{Distance in 2-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In 3-D, we can extend Pythagoras to the points $(x_1,\ y_1,\ z_1)$ and $(x_2,\ y_2,\ z_2)$:

$$\text{Distance in 3-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

E.g. 4 Find the distance between:
(a) $(1, 4)$ and $(13, 9)$ (b) $(1, 3, 4)$ and $(2, 8, 7)$

Working: (a) Label the points: $(1, 4)$ and $(13, 9)$
(x_1, y_1) (x_2, y_2)

$$\text{Distance in 2-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(13 - 1)^2 + (9 - 4)^2}$$
$$= \sqrt{12^2 + 5^2}$$
$$= 13 \text{ units}$$

(b) Label the points: $(1, 3, 4)$ and $(2, 8, 7)$
(x_1, y_1, z_1) (x_2, y_2, z_2)

$$\text{Distance in 3-D} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$= \sqrt{(2 - 1)^2 + (8 - 3)^2 + (7 - 4)^2}$$
$$= \sqrt{1^2 + 5^2 + 3^2}$$
$$= 5.92 \text{ units (3 s.f.)}$$

Video: Trigonometry in 3-D

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p587 E18.5 Qu 1-10
A*-G class textbook:	p546 E18.4 Qu 1-10
9-1 homework book:	p199 E18.5 Qu 1-6
A*-G homework book:	p152 E18.4 Qu 1-5

Homework book answers (only available during a lockdown)