UNIT 20 *Arithmetic: Fractions* Activities

Activities

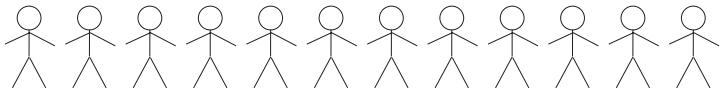
ACTIVITY 20.1 *Human Multiplication*

Illustrate multiplication of fractions by using groups of pupils.

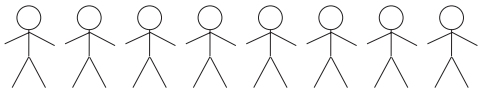
For example,

$$\frac{2}{3} \times \frac{1}{4}$$

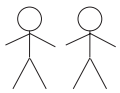
Start with 12 pupils:



Select $\frac{2}{3}$ of these:



Now select $\frac{1}{4}$ of these:



The result is $\frac{2}{12}$ of the number of pupils that you started with,

so $$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

In the same way, calculate:

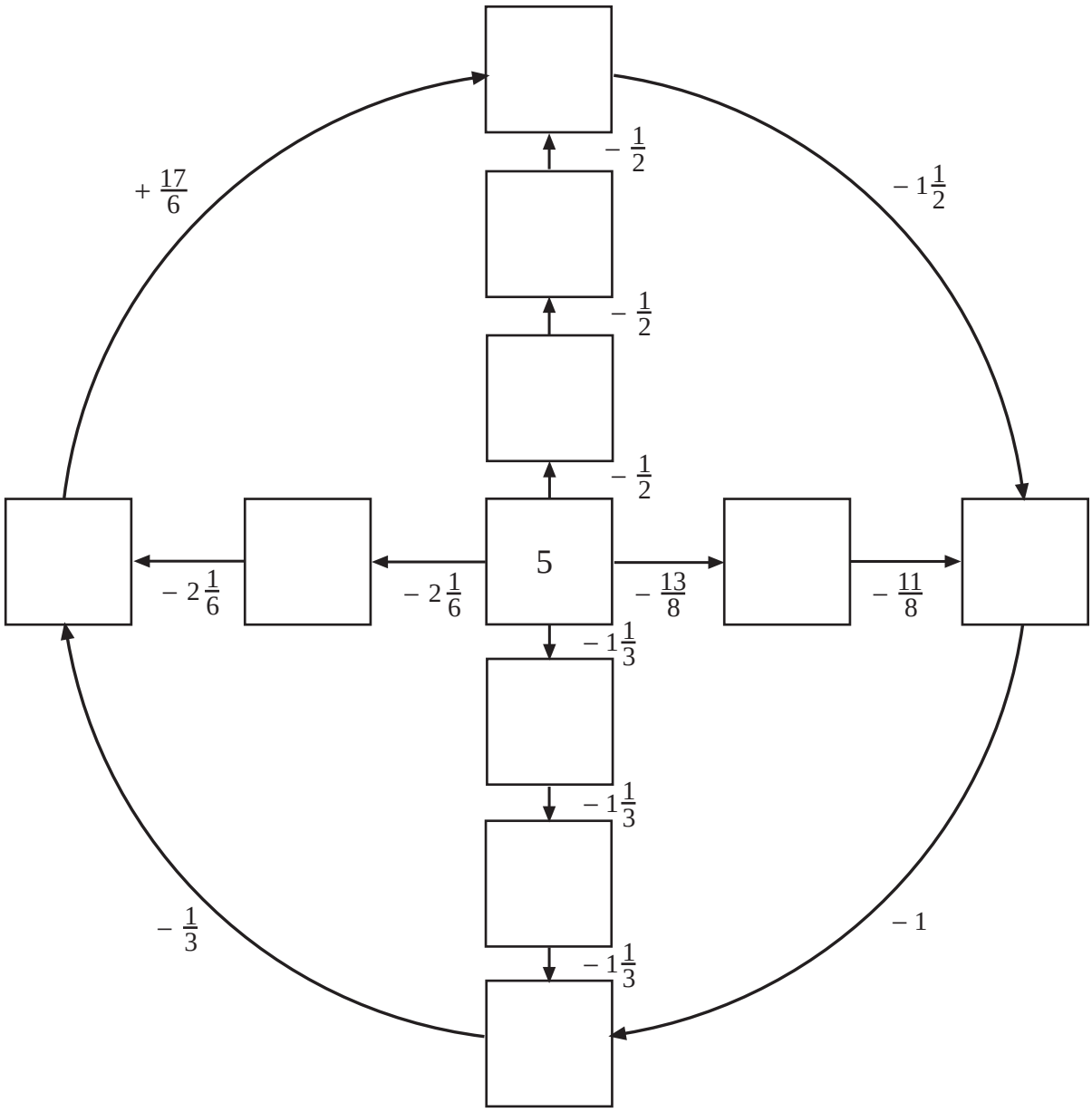
1. $\frac{1}{6} \times \frac{2}{3}$ starting with 18 pupils.

2. $\frac{1}{2} \times \frac{1}{4}$ starting with 8, 16 or 24 pupils.

3. $\frac{2}{5} \times \frac{2}{3}$ starting with 15 pupils.

ACTIVITY 20.2 *Adding and Subtracting Fractions*

Using the information given, write the correct number in each box on the diagram below:

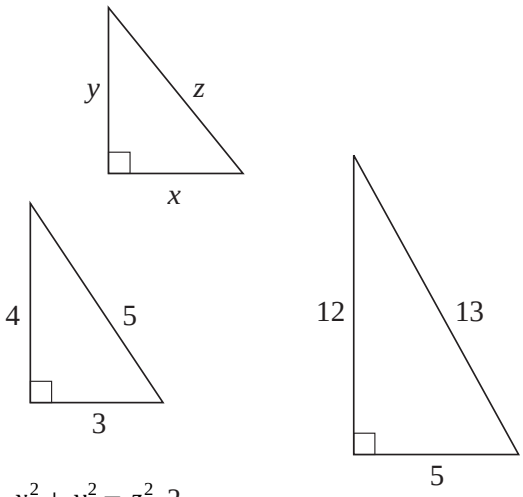


ACTIVITY 20.3 *Diophantine Equations*

Pythagoras' Theorem states that, for any right-angled triangle

$$x^2 + y^2 = z^2$$

You are probably familiar with the integer (whole number) solutions shown opposite:



1. What other integer solutions can you find to $x^2 + y^2 = z^2$?

In fact, equations which have integer solutions are known as *Diophantine* equations, after the Greek mathematician and philosopher, *Diophantos* of Alexandria. He lived in the 3rd century AD, and is credited with being the founder of modern algebra. The use of symbols to represent numbers was found in his published material, entitled *Arithmetic*.

He studied equations of the form:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$$

for some number, n, and where x and y are integers. For example, when $n = 6$,

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$

2. One possible solution is given by $x = 8,\ y = 24$. Check that these values give a solution for the equation above.

A second solution is obviously given by $x = 24,\ y = 8$, but there are many more.

3. Determine, in total, 17 solutions for this equation. Note that you can allow x or y to be a *negative* integer.

4. How many solutions can you determine for the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{8}$?

ACTIVITIES 20.1 - 20.2 *Notes and Solutions*

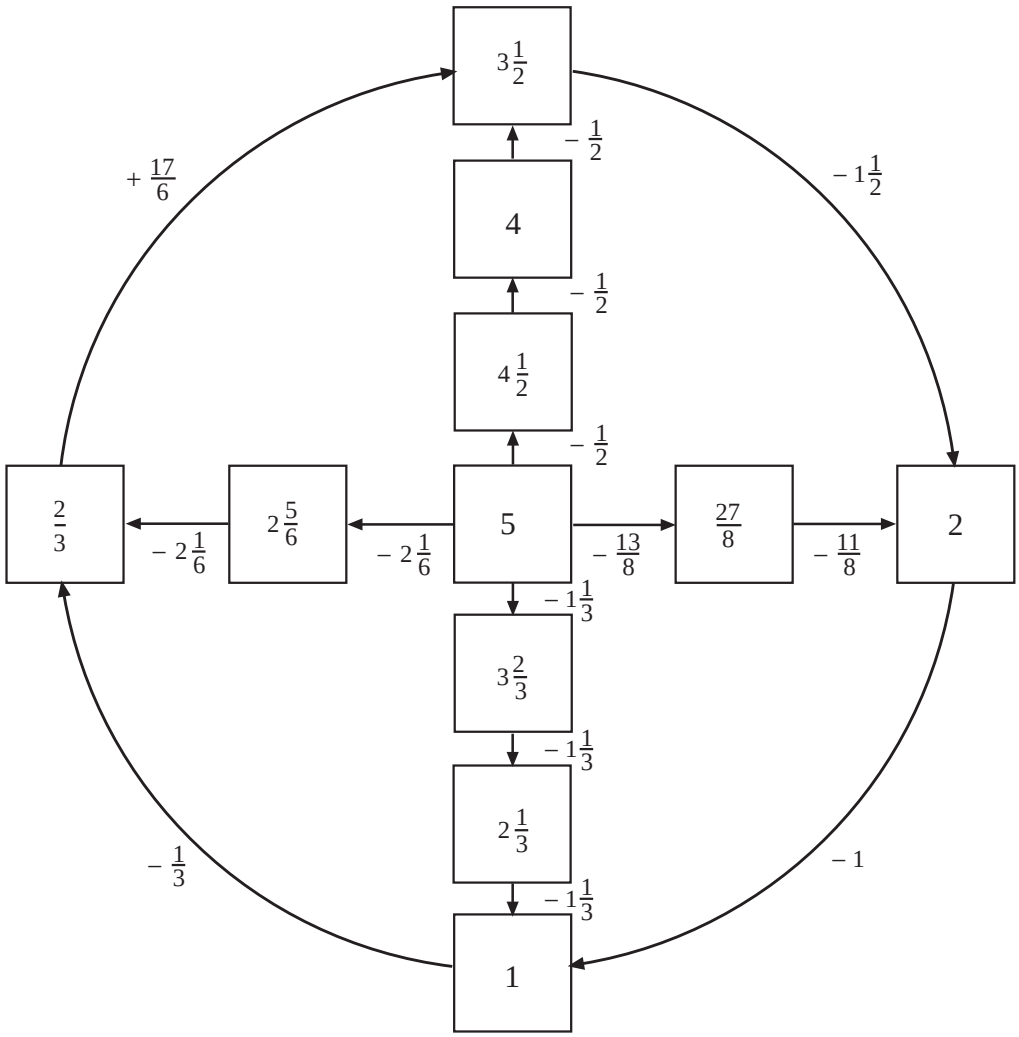
Notes and solutions are given only where appropriate.

20.1 1. $\frac{2}{3}$ of $18 = 12$; $\frac{1}{6}$ of $12 = 2$, i.e. $\frac{2}{18} = \frac{1}{9}$

2. $\frac{1}{4}$ of $8 = 2$; $\frac{1}{2}$ of $2 = 1$, i.e. $\frac{1}{8}$, etc.

3. $\frac{2}{3}$ of $15 = 10$; $\frac{2}{5}$ of $10 = 4$, i.e. $\frac{4}{15}$

20.2



ACTIVITIES 20.3 *Notes and Solutions*

20.3 1. There is an infinite number of solutions;
e.g. 9, 40, 41 or 6, 8, 10 (although this is really 3, 4, 5).

3.

x	12	8	24	3	-6	2	-3	4	-12	9	18	10	15	5	-30	7	42
y	12	24	8	-6	3	-3	2	-12	4	18	9	15	10	-30	5	42	7

4.

x	16	4	-8	12	24	6	-24	10	40	7	-56	9	72
y	16	-8	4	24	12	-24	6	40	10	-56	7	72	9

(13 solutions exist)