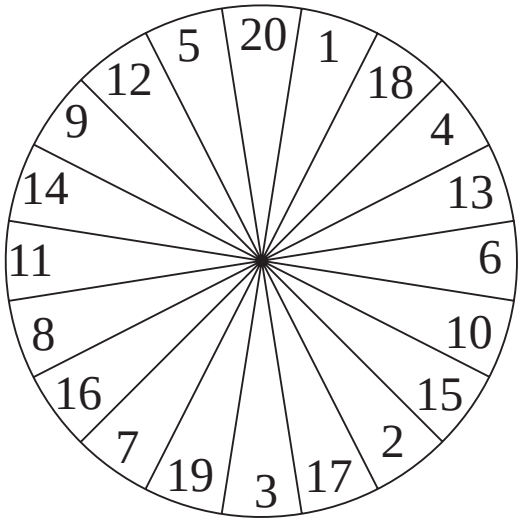
UNIT 21 *Probability of One Event* Activities

Activities

ACTIVITY 21.1 *Dart Board*

Here is a simplified diagram of a dart board, showing only the 20 sectors for the numbers 1-20 (and not doubles, trebles, 25 or 50).



Suppose you throw *one* dart at the board, and that if your dart misses the board you have another throw until you are successful.

1. Assuming that your dart is equally likely to land in any sector of the board, what is the probability of obtaining, with *one* dart:

   (a) 20,

   (b) an even number,

   (c) 18, 19 or 20,

   (d) a number which is a multiple of 3,

   (e) a prime number?

Extension

1. You throw the dart many times: you are aiming for the 20, but your dart is equally likely to hit either of the sectors on each side (5 or 1). What is the average score expected?

2. You throw the dart many times without aiming at any particular sector. What is the average score that you would expect to get?

ACTIVITY 21.2 *Simple Lottery*

The activity is a lottery-type game for the whole class.

In this lottery, each pupil chooses a lottery 'card' that has two different digits on it from the digits

1 2 3 4 5 6

(For example, (1, 2) or (1, 6), etc.).

An unbiased dice is thrown twice (repeating the second throw if identical digits are thrown), giving the winning pair of numbers.

Check how many pupils in your class:

(a) get *both* digits the same as the winning pair of numbers,

(b) get *just one* digit the same as one of the winning numbers.

Repeat the game several times, and use the data to estimate the probability of:

(a) winning, (b) getting one digit the same.

These should be reasonably close to the theoretical probabilities.

1. List all the possible outcomes, i.e. pairs of digits (in any order).

2. As each outcome is equally likely, what is the probability of winning if you have one 'card' ?

3. Suppose the winning numbers are 1 and 2. How many outcomes have only 1 or 2 (but not both digits) on them? What is the probability of getting just one digit the same?

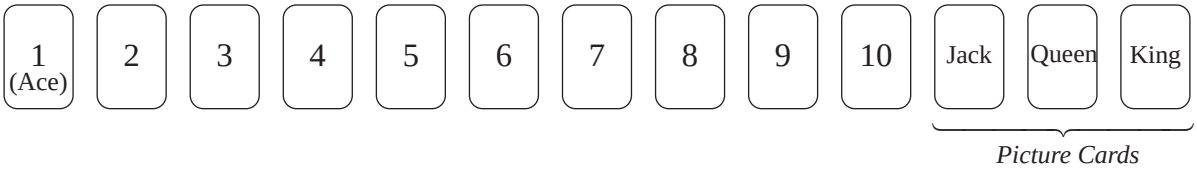
Extension

1. Repeat the activity with, for example, the numbers 1-10.

2. Suppose the 'card' costs £1, and the prizes are £5 for winning and £2 for matching just one number. Is it worth 'buying' a card?

ACTIVITY 21.3 *Playing Cards*

In a pack of playing cards, there are 13 cards (Ace, 2, 3, . . . , 10, Jack, Queen, King) in each of 4 suits (Diamonds ◇, Hearts ♡, Clubs ♣, Spades ♠).



1. How many cards are there, in total, in a pack of playing cards?

2. What is the probability, when taking a card at random from a complete pack of playing cards, of obtaining:

   (a) the Queen of Hearts,

   (b) a Heart,

   (c) any King

   (d) the Ace (1) of Hearts *or* the Ace of Diamonds,

   (e) a Heart *and* a picture card,

   (f) any picture card ?

Extension

You are dealt two cards at random, one at a time, from a complete pack of playing cards. What is the probability that, *given* that the first card dealt is the *Ace of Hearts*, the second card is:

   (a) a Heart,

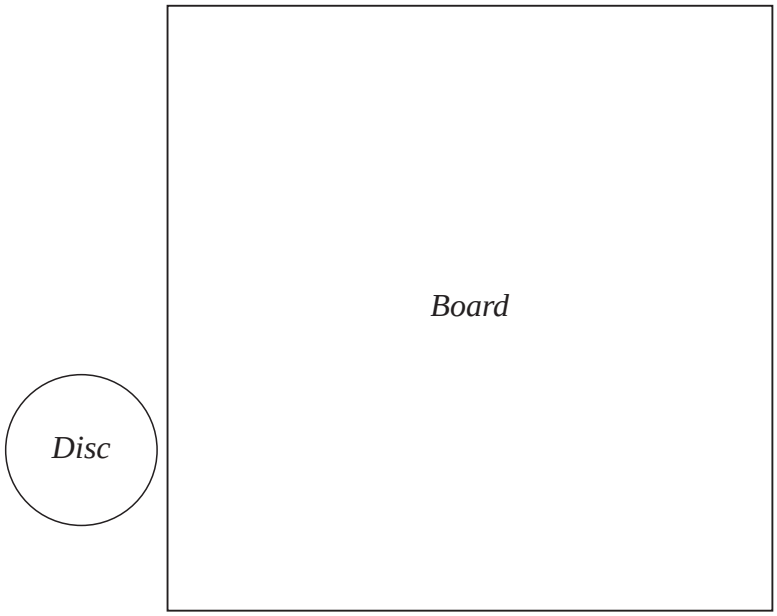
   (b) a Diamond,

   (c) an Ace,

   (d) a picture card?

ACTIVITY 21.4 *Board Game*

An entry fee of 10p is payable for this game of chance, where you throw a 1 cm radius disc onto a 8 cm × 8 cm board. If the whole of the disc lands completely on the board, you win 20p. If the disc lands astride the edge then you lose your money. If you miss totally, you have another go until you get the disc on, or partly on, the board.



The question must now be answered:

"Is the game worth playing?".

1. What are the dimensions of the *square* inside which the centre of the disc must lie if it is to be in a winning position?
2. What is the area of this *square*?
3. What is the area of the *board*?
4. What is the probability of winning, assuming that the disc lands somewhere on the board?
5. What would you expect to win, on average, if you played this game many times? Is it worth playing?

Extension

Analyse a similar game, but using different dimensions, for example:

(i) 6 cm × 8 cm board,

and (ii) 2 cm radius disc on the board above.

ACTIVITIES 21.1 - 21.3 *Notes and Solutions*

Notes and solutions are given only where appropriate.

21.1 1. (a) $\frac{1}{20}$ (b) $\frac{1}{2}$ (c) $\frac{3}{20}$ (d) $\frac{3}{10}$ (e) $\frac{2}{5}$

Extension

1. $\frac{26}{3} \approx 8.7$ 2. 10.5 (average of 1, 2, 3, . . ., 20)

21.2 1. (1, 2) (2, 3) (3, 4) (4, 5) (5, 6)
(1, 3) (2, 4) (3, 5) (4, 6)
(1, 4) (2, 5) (3, 6)
(1, 5) (2, 6)
(1, 6)

2. $\frac{1}{15}$

3. 8, $\frac{8}{15}$

Extension

2. Expected profit = Expected winnings – Stake money (cost of card)

$$= £\left(5 \times \frac{1}{15} + 2 \times \frac{8}{15} - 1\right)$$

$$= £\left(\frac{21}{15} - 1\right)$$

$$= £\frac{2}{5} = 40\text{p}$$

Yes; it is worth buying a ticket.

21.3 1. 52

2. (a) $\frac{1}{52}$ (b) $\frac{13}{52} = \frac{1}{4}$ (c) $\frac{4}{52} = \frac{1}{13}$ (d) $\frac{2}{52} = \frac{1}{26}$
(e) $\frac{3}{52}$ (f) $\frac{12}{52} = \frac{3}{13}$

Extension

(a) $\frac{12}{51}$ (b) $\frac{13}{51}$ (c) $\frac{3}{51} = \frac{1}{17}$ (d) $\frac{11}{51} = \frac{4}{17}$

ACTIVITIES 21.4 *Notes and Solutions*

21.4 1. $6\text{ cm} \times 6\text{ cm}$

2. 36 cm^2

3. 64 cm^2

4. $p = \dfrac{36}{64} = \dfrac{9}{16}$

5. Expected profit per game $= 20 \times \dfrac{9}{16} - 10 = \dfrac{20}{16} = 1.25\text{p}$

Yes – you would expect to win 1.25p, on average, per game.

Extension

(i) For 6 cm × 8 cm boards, $p = \dfrac{24}{48} = \dfrac{1}{2}$

average profit $= 20 \times \dfrac{1}{2} - 10 = 0\text{p}$ per game

(ii) For 2 cm radius disc, and 8 cm × 8 cm board,

$$p = \frac{16}{64} = \frac{1}{4}$$

average profit $= 20 \times \dfrac{1}{4} - 10 = -5\text{p}$ per game;

it is *not* worth playing.