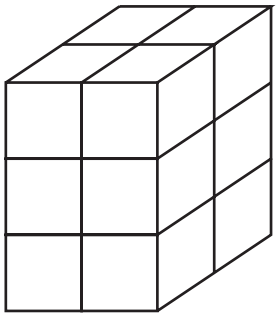
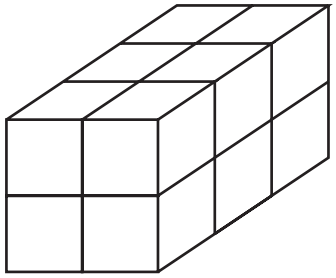
UNIT 22 *Volume* Activities

Activities

ACTIVITY 22.1 *Making Cuboids*

1. You have 12 cubes, each with sides 1 cm long. How many different cuboids can you make using all the cubes for each one?





Two are shown here, but these are essentially the same, and could be described as $3 \times 2 \times 2$ cuboids. There are 3 other different cuboids that can be made from 12 cubes. Describe them.

2. How many different cuboids can you make using:

   (a) 16 cubes,

   (b) 10 cubes,

   (c) 7 cubes,

   (d) 5 cubes,

   (e) 9 cubes ?

3. Without drawing them or using cubes, decide how many different cuboids you could make using the following numbers of cubes:

   (a) 17,

   (b) 8,

   (c) 20,

   (d) 49,

   (e) 15.

Extension

Can you determine a general result which gives you the number of different cuboids it is possible to make possible using *any* number of cubes?

(*Hint*: write each number as a product of its prime factors and look at the sum of the powers for each number.)

ACTIVITY 22.2 *Cereal Density*

Collect data from as many cereal boxes as possible, when you next visit a supermarket.

Determine the *volume* that each one contains by measuring the box, and the *mass* of its contents (printed on box). Calculate the *density* of the contents of each box, and complete a table like the one below:

Cereal	*Mass*	*Volume*	*Density*

Comment on your results; for example, decide which is the *most dense* cereal, and which the *least dense*.

You might like to consider the following questions:

1. Do *all* brands of the same type of cereal have the *same* density?

2. Do large packets of cereal have the same density as smaller packets of the same cereal?

Extension

In each case, determine the cost per gm of the cereal. Are the larger boxes better value than the smaller ones?

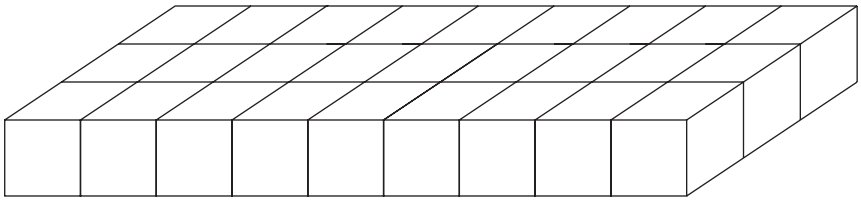
ACTIVITY 22.3 *Surface Area of Cuboids*

If you are given 27 cubes, each with sides 1 cm long,

(a) describe the 3 cuboids you can make, using all 27 of the cubes,

(b) which of these cuboids has the *smallest* surface area?

For example, this cuboid is made from 27 cubes:



Surface area $= 9 + 9 + 3 + 3 + 27 + 27$

$= 78\ \text{cm}^2$

Record your results in a table:

Cuboid	*Surface Area* (cm^2)
$9 \times 3 \times 1$	78

1. Describe cuboids that have the smallest surface area that can be made from:

   (a) 8 cubes, each with sides 1 cm long,

   (b) 125 cubes, each with sides 1 m long,

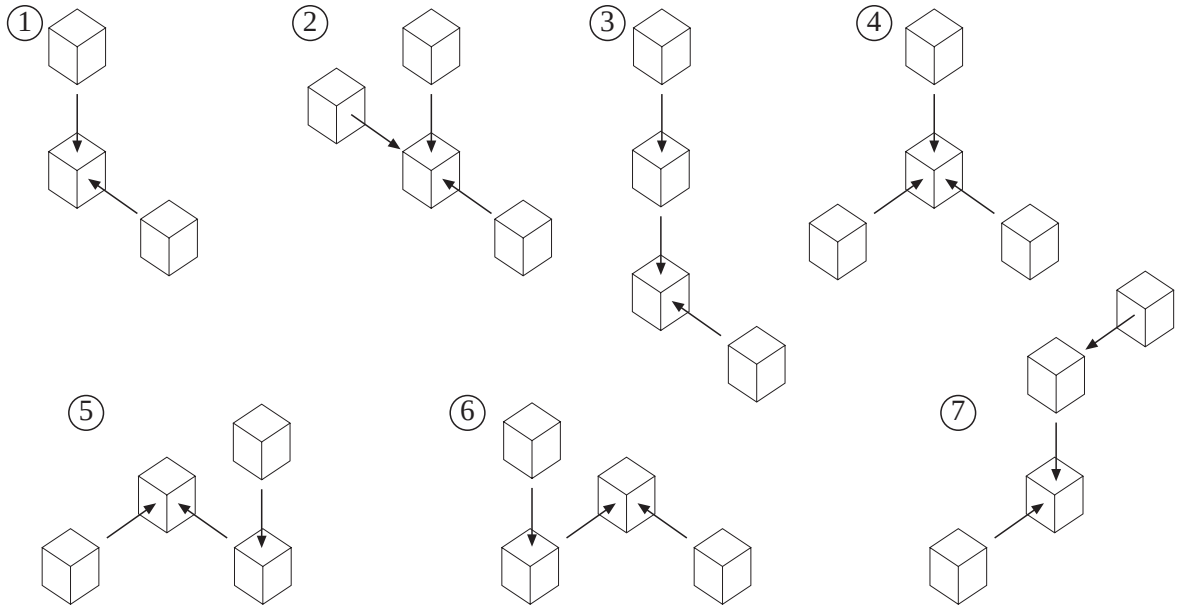
   (c) 32 cubes, each with sides 1 cm long,

   (d) 40 cubes, each with sides 1 cm long.

2. Describe how to calculate the smallest surface area for a cuboid made from a certain number of cubes.

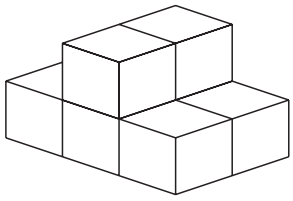
ACTIVITY 22.4 *Soma Puzzle*

The Danish puzzle expert *Piet Hein* invented the activity below, consisting of 7 pieces – one piece contains 3 unit cubes, whilst all the others include 4 unit cubes. The method of construction is shown below:

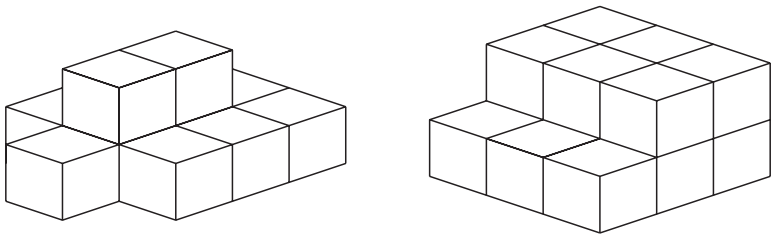


These 7 pieces combine together (in 240 different ways!) to form a $3 \times 3 \times 3$ cube . Before attempting the cube construction, try the two problems below. First, though, make all the 7 pieces, using 27 cubes.

1. Combine any two pieces to form this structure:



2. Use all seven pieces to make the two shapes below:



Extension

Make a $3 \times 3 \times 3$ cube, using all 7 pieces.

ACTIVITY 22.1 *Notes and Solutions*

Notes and solutions are given only where appropriate.

22.1 1. $12 \times 1 \times 1$, $6 \times 2 \times 1$, $4 \times 3 \times 1$

2. (a) 4, $16 \times 1 \times 1$, $8 \times 2 \times 1$, $4 \times 4 \times 1$, $4 \times 2 \times 2$

(b) 2, $10 \times 1 \times 1$, $5 \times 2 \times 1$

(c) 1, $7 \times 1 \times 1$

(d) 1, $5 \times 1 \times 1$

(e) 2, $9 \times 1 \times 1$, $3 \times 3 \times 1$

3. (a) 1, $17 \times 1 \times 1$

(b) 3, $8 \times 1 \times 1$, $4 \times 2 \times 1$, $2 \times 2 \times 2$

(c) 4, $20 \times 1 \times 1$, $10 \times 2 \times 1$, $5 \times 4 \times 1$, $5 \times 2 \times 2$

(d) 2, $49 \times 1 \times 1$, $7 \times 7 \times 1$

(e) 2, $15 \times 1 \times 1$, $5 \times 3 \times 1$

Extension

General results can be deduced from the factorisation into prime numbers,,

e.g. $17 = 17^1 \Rightarrow$ 1 cuboid

$8 = 2^3 \Rightarrow$ 3 cuboids

$20 = 2^2\ 5^1 \Rightarrow 2 + 1 = 3$ cuboids

$49 = 7^2 \Rightarrow$ 2 cuboids

$15 = 3^1\ 5^1 \Rightarrow 1 + 1 = 2$ cuboids

Note that this does not always give the exact answer, but it does give a lower bound. In some cases, the lower bound is the exact answer, but note, for example, that:

(i) $100 = 2^2\ 5^2 \Rightarrow 2 + 2 = 4$ cuboids, but there are 6 possible cuboids,

(ii) $128 = 2^7 \Rightarrow$ 7 cuboids, but there are, in fact, 8 possible cuboids.

ACTIVITIES 22.3 - 22.4 *Notes and Solutions*

22.3

Cuboid	*Surface Area* (cm^2)
$9 \times 3 \times 1$	78
$27 \times 1 \times 1$	110
$3 \times 3 \times 3$	54

27 cubes

Minimum for $3 \times 3 \times 3$.

1. (a)

Cuboid	*Surface Area* (cm^2)
$8 \times 1 \times 1$	34
$4 \times 2 \times 1$	28
$2 \times 2 \times 2$	24

8 cubes.

Minimum for $2 \times 2 \times 2$.

(b)

Cuboid	*Surface Area* (cm^2)
$125 \times 1 \times 1$	502
$25 \times 5 \times 1$	310
$5 \times 5 \times 5$	150

125 cubes.

Minimum for $5 \times 5 \times 5$.

(c)

Cuboid	*Surface Area* (cm^2)
$32 \times 1 \times 1$	130
$16 \times 2 \times 1$	100
$8 \times 4 \times 1$	88
$8 \times 2 \times 2$	72
$4 \times 4 \times 2$	64

32 cubes.

Minimum for $4 \times 4 \times 2$.

(d)

Cuboid	*Surface Area* (cm^2)
$40 \times 1 \times 1$	162
$20 \times 2 \times 1$	124
$10 \times 4 \times 1$	108
$10 \times 2 \times 2$	88
$8 \times 5 \times 1$	106
$5 \times 4 \times 2$	76

40 cubes.

Minimum for $5 \times 4 \times 2$.

2. Minimum for shape that is a cube or closest to a cube.

22.4 1. Note that 8 unit cubes are needed for this shape, so that the No. 1 piece (which has 3 unit cubes) is not suitable.

2. The shape on the left uses pieces 2, 4 and 5. Note that it is possible, using other pieces, to form the shape on the left, and then be unable to make the shape on the right.