

Lesson 14 – HCF and LCM with Prime Factors

Starter

- 1) Write 84 as the product of prime factors
- 2) Find the HCF of 24 and 36
- 3) Find the LCM of 12 and 40
- 4) Find two numbers that have a HCF of 8

Starter Answers

- 1) $2^2 \times 3 \times 7$ 2) 12 3) 120 4) e.g. 8 and 16

Highest Common Factor

We can use prime factorisation to find the factors of a number. So, we can also use this to find the common factors between two numbers.

Example 1

Find the highest common factor of 16 and 24.

First write 16 and 24 as the product of prime factors:

$$16 = 2 \times 2 \times 2 \times 2$$

$$24 = 2 \times 2 \times 2 \times 3$$

We need to look for the highest factor that both numbers have in common. So we look for the highest factor we can make using the prime factors in both products.

$$16 = \boxed{2 \times 2 \times 2} \times 2$$

$$24 = \boxed{2 \times 2 \times 2} \times 3$$

So, the highest factor they have in common is $2 \times 2 \times 2 = 8$

$$\text{HCF} = 8$$

Example 2

Find the highest common factor of 270 and 600

Write 270 and 600 as the product of prime factors:

$$270 = \boxed{2} \times \boxed{3} \times 3 \times 3 \times \boxed{5}$$

$$600 = \boxed{2} \times 2 \times 2 \times \boxed{3} \times \boxed{5} \times 5$$

The highest factor in common is: $2 \times 3 \times 5 = 30$

$$\text{HCF} = 30$$

Your go

Use prime factorisation to find the highest common factor of:

- 1) 18 and 27
- 2) 64 and 96
- 3) 300 and 504
- 4) 108 and 450
- 5) 1890 and 7938

Answers

- 1) 9 2) 32 3) 12 4) 18 5) 378

Lowest Common Multiple

We can also find the lowest common multiple using prime factors.

Example 3

Find the lowest common multiple of 12 and 24

Write 12 and 24 as the product of prime factors:

$$12 = 2 \times 2 \times 3$$

$$24 = 2 \times 2 \times 2 \times 3$$

Find the HCF = $2 \times 2 \times 3 = 12$

We now need to multiply by the remaining prime factors to ensure that the number is a multiple of both 12 and 24. The only factor remaining is 2, so:

$$12 \times 2 = 24$$

$$\text{LCM} = 24$$

Example 2

Find the lowest common multiple of 126 and 540

Write as the product of prime factors:

$$126 = 2 \times 3 \times 3 \times 7$$

$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

Find the HCF = $2 \times 3 \times 3 = 18$

Multiply by the remaining prime factors = $18 \times 7 \times 2 \times 3 \times 5 = 3780$

$$\text{LCM} = 3780$$

Your go

Use prime factorisation to find the lowest common multiple of:

- 1) 18 and 27
- 2) 64 and 96
- 3) 300 and 504
- 4) 108 and 450
- 5) 1890 and 7938

Answers

- 1) 54 2) 192 3) 12600 4) 2700 5) 39690

Example 3

Two numbers, A and B, are given by $A = 2^2 \times 3^3 \times 5$, $B = 2^3 \times 3 \times 5^3 \times 7$

- a) Find the HCF
- b) Find the LCM

This time the numbers have already been written as the product of prime factors

$$A = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \quad B = 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7$$

a) $HCF = 2 \times 2 \times 3 \times 5 = 60$

b) $LCM = 60 \times 3 \times 3 \times 2 \times 5 \times 5 \times 7 = 189000$