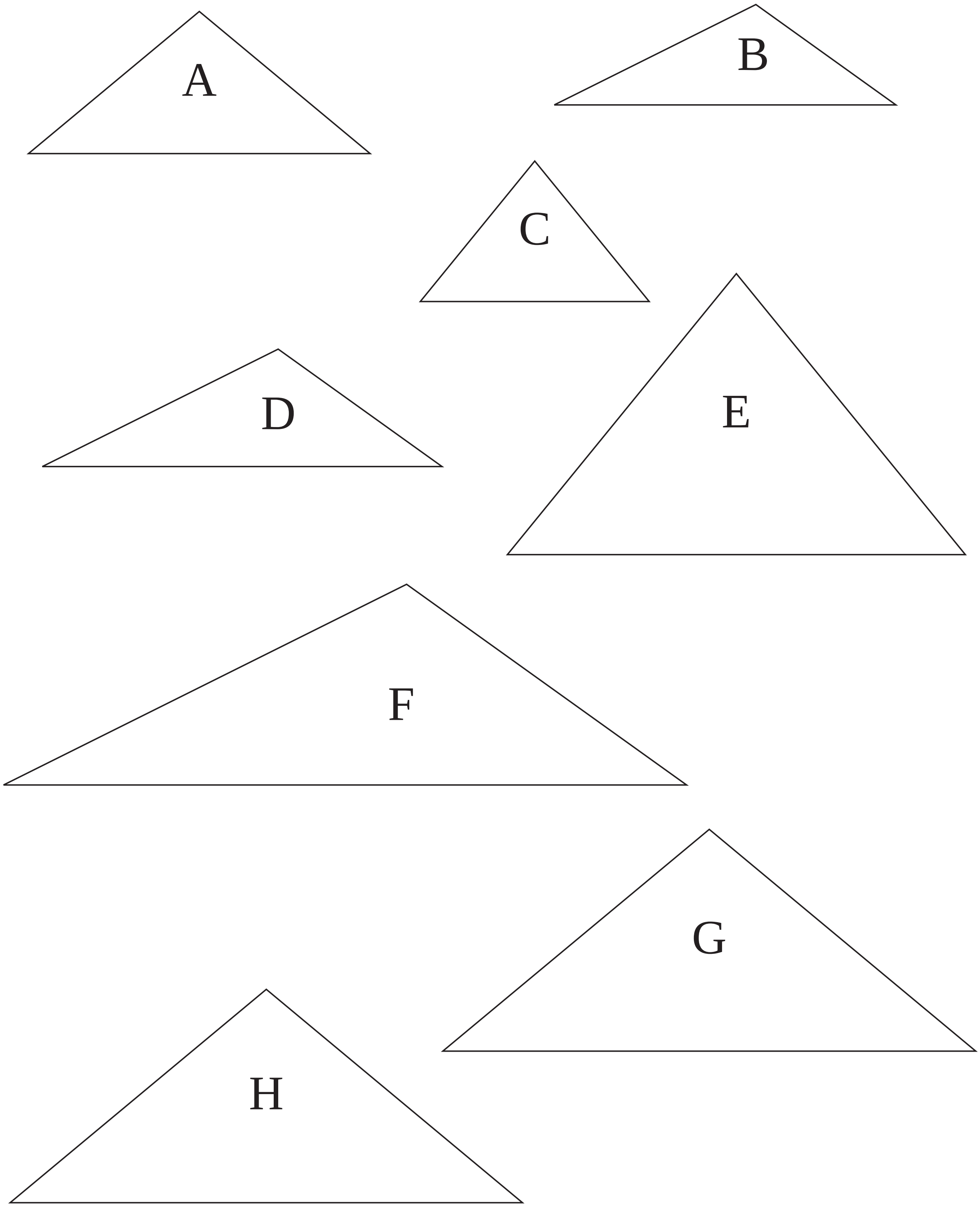
UNIT 19 *Similarity* Activities

Activities

ACTIVITY 19.1 *Similar Shapes*

Copy and cut out the triangles shown. Decide which are similar.

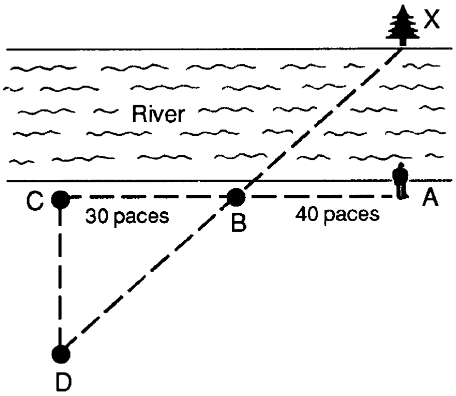


ACTIVITY 19.2 *How Far Away?*

Pace Length Before you attempt the tasks below, you must find the length of your pace. To do this, it is best to walk at least 10 paces, measure the distance and divide by the number of paces.

DISTANCE

It is useful to be able to judge distances, and it is often not as difficult as you might think. Suppose, for example, you want to estimate the distance across a river. You can follow the procedure below:



- Find a landmark, say X, immediately opposite from you, A, on the other bank.
- Walk along your side of the river for, say, 40 paces, and mark the point with a stick, B.
- Walk on 30 paces to C, and walk inland to a point D so that DBX are in a straight line.
- Count the number of paces taken between C and D.

1. What can you say about triangles DCB and XAB ?

2. What is the ratio $\frac{XA}{CD}$? XA can now be found using this ratio.

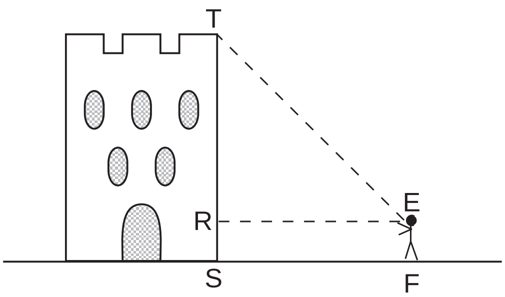
HEIGHT

There is a similar method of estimating the height of a building or tower. You follow the procedure below, but first you need to make a *sight gauge*.

To Make a Sight Gauge

Fold a strip of card as shown opposite, to give a $45°$ angle.

- Starting at the foot of the tower, count the number of paces as you walk away.
- Stop walking when, looking along your sight gauge, you can see the top of the tower. (Make sure that you keep the gauge level.)



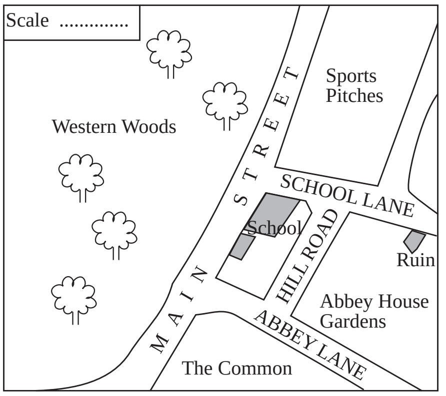
3. Why is $TR = RE$?

4. Explain why the height of the tower $\approx RE + \text{your height}$.

Extension

1. Try out both these methods, and judge their accuracy.
2. Discuss other ways of estimating distances.

ACTIVITY 19.3 *Using Maps to Estimate Areas*



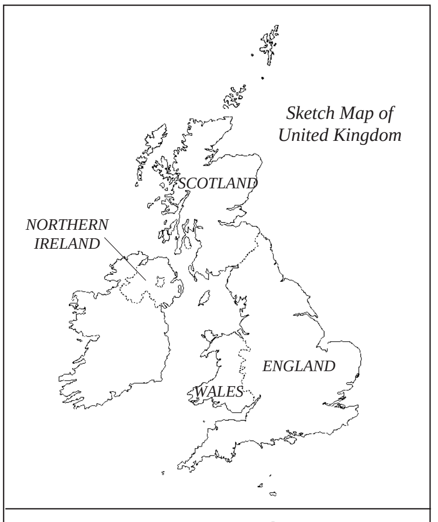
Obtain copies of a detailed local map of your area.

1. Use the map and its scale to calculate, for example, the:
   (a) area of school grounds,
   (b) area of local sports pitches,
   (c) area of interesting local features.

   Calculate the areas in cm^2, m^2 and km^2 as appropriate.

2. Check your results to question 1 by actually measuring the area being estimated from the map. (You could 'pad' the area (on foot) or use an extended measure which you could probably obtain from your school PE department.)

Extension



Use a map from an atlas to estimate the area of:
(a) England,
(b) Wales,
(c) Scotland,
(d) the county you live in.

Check your estimate with published data, which can be found on the internet at

www.ex.ac.uk/cimt/data/datalist.htm

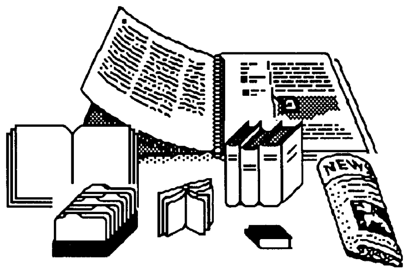
and in publications such as

'The Guinness World Data Book'
ISBN 0-85112-960-9 (p 176).

Local information can be obtained from your local authority or public library.

Information is also available on the internet at
http://www.odci.gov/cia/publications/factbook/index.html

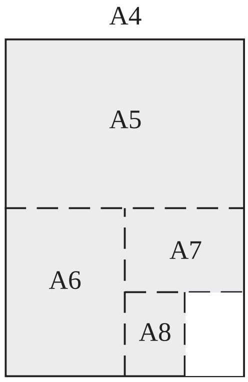
ACTIVITY 19.4 *Paper Sizes*



The 'A' series of interational paper sizes consists of the following sizes:

A0, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10.

You are probably familiar with A4 (the usual size for photocopier machines). A5 is half the size of A4, A6 is half the size of A5, etc.



1. Measure the width and length of an A4 sheet of paper to the nearest millimetre.

2. Cut out from the A4 sheet,

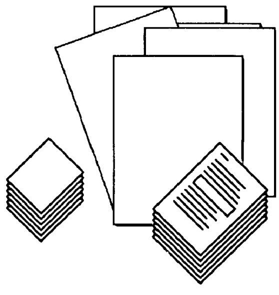
   A5, A6, A7, A8, A9 and A10

   sizes of paper. Measure the width and length for each size.

3. For each size, note the ratio of length to width. What do you notice?

If all the paper sizes are similar, then we will see that there is only *one* possible ratio r, for length to width.

If the dimensions of one size are width, a and length, ra, then the dimensions of the next smaller size are width, $\frac{ra}{2}$ and length, a.



4. Write down the ratio $\left(\frac{\text{length}}{\text{width}}\right)$ for the two sizes.

   If these two ratios are equal, show that $r^2 = 2$.

So, given just *one* dimension, for example, that the width of A4 is 210 mm, we can deduce the dimensions of all other sizes.

5. Deduce that the dimensions of A10 are $\frac{210}{\left(\sqrt{2}\right)^5}$ by $\frac{210}{\left(\sqrt{2}\right)^6}$.

   Check this result with your experimental data for A10.

6. Deduce the dimensions of A0.

Extension

The dimensions of 'A' size paper are mostly awkward numbers (due to the $\sqrt{2}$ multiplier). Start with a $1\ \text{m}^2$ area of paper, and divide up the sheet to give a series of different sizes which would be practical to use.

ACTIVITIES 19.1 - 19.3 *Notes and Solutions*

Notes and solutions given only where appropriate.

19.1 One method of deducing the answer is to measure the three sides of each triangle, list them and check which are in the same ratio.

Triangles A, G and H are similar.

Triangles B, D and F are similar.

Triangles C and E are similar.

Alternatively, the angles of each triangle can be measured and, where all 3 angles are the same, the triangles are similar.

19.2 Pupils will need to know the length of their 'pace'.

- You might find it interesting to know that Roman soldiers could march 60 000 paces a day.
- *Leonardo Da Vinci* thought that in well-proportioned persons, their pace lengths should be about half their height.

1. The triangles are similar.

2. $\frac{XA}{CD} = \frac{BA}{BC} = \frac{40}{30}$, so $XA = \frac{4}{3} \times CD$

3. $TR = RE$ since TRE is a right-angled isosceles triangle $\left(\text{angle } RTE = \text{angle } RET = 45°\right)$.

4. The height of the tower $= TR + RS = RE +$ height of observer.

Extension Sight can be used to estimate distances but it is probably not a very accurate method.

Sound can sometimes be used. Sound travels at a speed of 333 ms^{-1}, so if there is some way to see an activity and to time how long it takes to *hear* the activity, we can use the formula

$$\text{distance (m)} = \text{time (s)} \times 330$$

For example, a lightning flash can be seen almost instantaneously, but it often takes some time for the thunder clap to be heard. (Every 5-second interval equates to about 1 mile distance.)

Another example is the time taken between seeing a starting pistol being fired (noting the smoke) and hearing the sound.

19.3 *Extension* (a) England: 130 439 square kilometres (50 363 square miles)

(b) Wales: 20 758 square kilometres (8015 square miles)

(c) Scotland: 78 789 square kilometres (30 420 square miles)

ACTIVITY 19.4 *Notes and Solutions*

19.4 For this practical exercise you will need paper of various sizes (A3, A4 and A5), scissors and rulers for the class. The actual dimensions for 'A' sizes of paper are given below.

Size	*Length* (mm)	*Width* (mm)
A0	1189	841
A1	841	594
A2	594	420
A3	420	297
A4	297	210
A5	210	140
A6	148	105
A7	105	74
A8	74	52
A9	52	37
A10	37	26

1. and 2. See above.

3. Constant ratio of about 1.4.

4. $\frac{ra}{a} = \frac{a}{\frac{1}{2}(ra)} \Rightarrow \frac{1}{2}(ra)^2 = a^2 \Rightarrow r^2 = 2$ and $r = \sqrt{2}$

5. A4 : $\sqrt{2}\,a$ by a A5 : a by $\frac{a}{\sqrt{2}}$ A6 : $\frac{a}{\sqrt{2}}$ by $\frac{a}{\left(\sqrt{2}\right)^2}$

A7 : $\frac{a}{\left(\sqrt{2}\right)^2}$ by $\frac{a}{\left(\sqrt{2}\right)^3}$, etc.

A10 : $\frac{a}{\left(\sqrt{2}\right)^5}$ by $\frac{\text{a}}{\left(\sqrt{2}\right)^6} \Rightarrow \frac{210}{4\sqrt{2}}$ by $\frac{210}{8}$

i.e. 37.12 by 26.25 which approximates to 37 by 26.

6. A3 : $\left(\sqrt{2}\right)^2 a$ by $\sqrt{2}\,a$ A4 : $\left(\sqrt{2}\right)^3 a$ by $\left(\sqrt{2}\right)^2 a$ A1 : $\left(\sqrt{2}\right)^4 a$ by $\left(\sqrt{2}\right)^3 a$

A0 : $\left(\sqrt{2}\right)^5 a$ by $\left(\sqrt{2}\right)^4 a \Rightarrow \left[\left(4\sqrt{2}\right) \times 210\right]$ by (4×210) is 1188 by 840.

(This is slightly different from the table above, due to inaccuracies brought in by the use of $\sqrt{2}$ as a multiplier.)

Extension

One possible solution is to divide the sheet into 500 50 mm × 40 mm rectangles. The 50 mm × 40 mm rectangle becomes the basic shape from which paper sizes can be made.