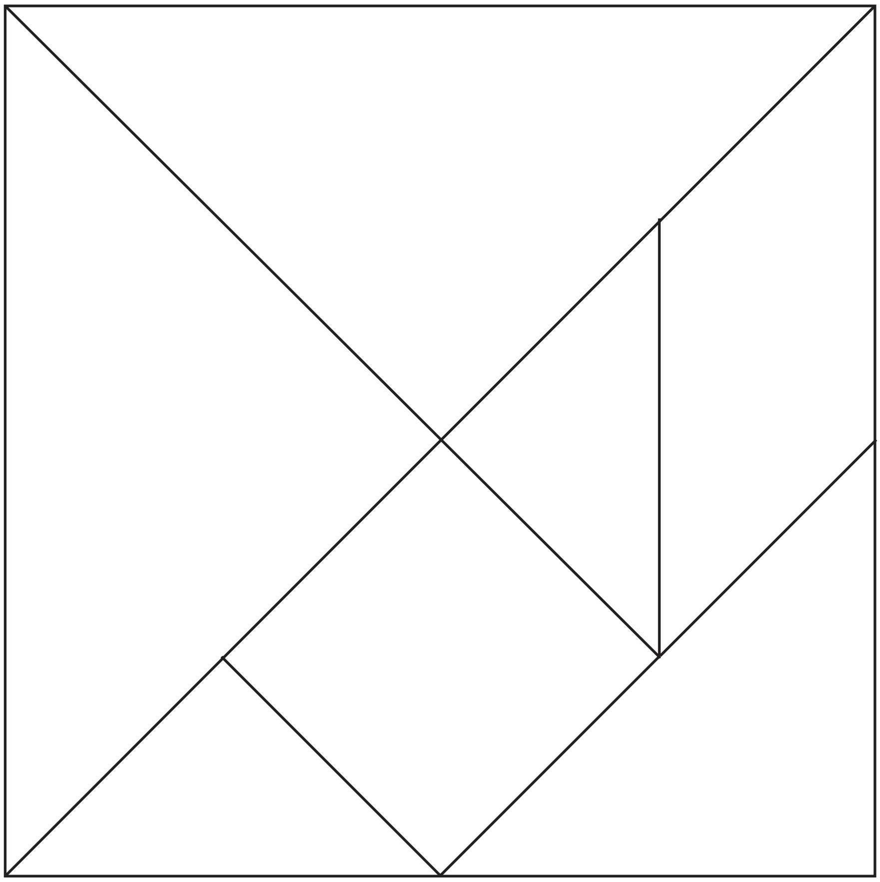
UNIT 6 *Nets and Surface Area* Activities

Activities

6.1	Tangram
6.2	Square-based Oblique Pyramid
6.3	Pyramid Packaging
6.4	Make an Octahedron
6.5.1	Klein Cube
6.5.2	" "
6.5.3	" "
6.6	Euler's Formula
	Notes and Solutions (3 pages)

ACTIVITY 6.1 *Tangram*

Cut out the following square into 7 shapes:



The puzzle is known as a *tangram*, and is a very old Chinese toy.

Rearrange the pieces you have cut out to form:

(a) a square from two triangles, and then change it to a parallelogram;

(b) a rectangle using three pieces, and then change it into a parallelogram;

(c) a trapezium with three pieces;

(d) a parallelogram with four pieces;

(e) a trapezium from the square, parallelogram and the two small triangles;

(f) a triangle with three pieces;

(g) a rectangle with all seven pieces.

Finally, put the pieces back together to form the original square.

ACTIVITY 6.2 *Square-based Oblique Pyramid*

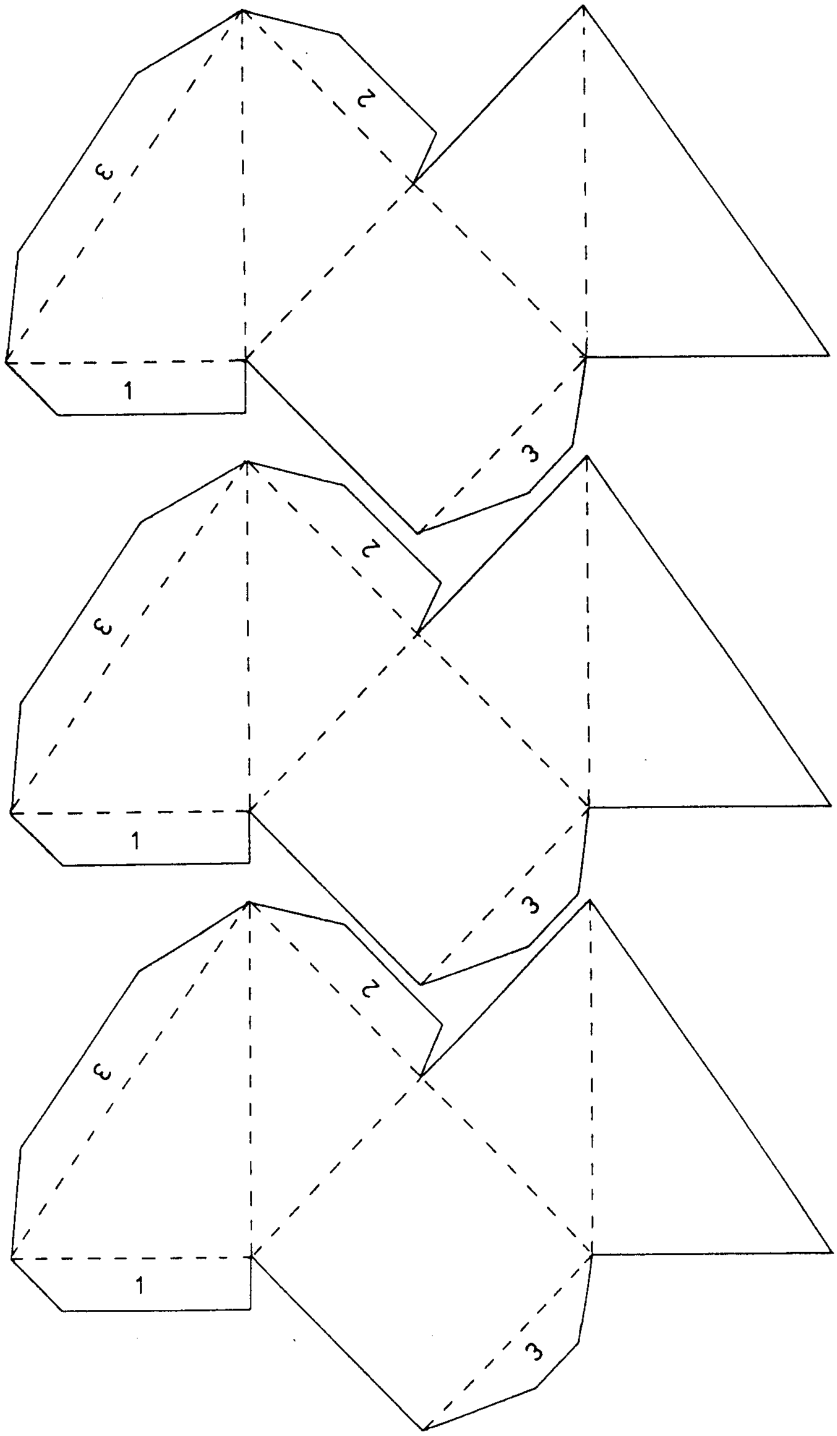
Following the instructions, make three square-based oblique pyramids from the identical shapes below.

Instructions *Cut out along the solid lines; score and fold along the broken lines.*

Roughly fold each shape into position to see how it will look.

Glue the tabs in the order they are numbered (where tabs have the same numbers, glue them at the same time).

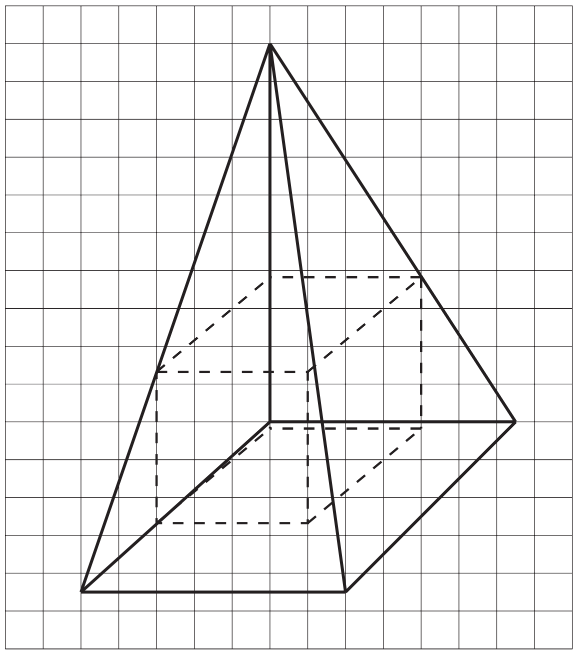
Now put your three shapes together to form a cube.



ACTIVITY 6.3 *Pyramid Packaging*

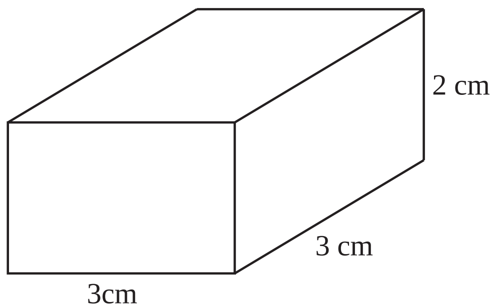
As part of a special promotion, boxes of sweets are to be packed inside special pyramid-shaped containers.

The following diagram shows just one way of fitting a 2 cm cube inside a pyramid:



1. Design and construct a pyramid that a 2 cm cube will just fit inside.

2. Design and construct a pyramid that a $3\text{ cm} \times 3\text{ cm} \times 2\text{ cm}$ cuboid will just fit inside.



ACTIVITY 6.4 *Make an Octahedron*

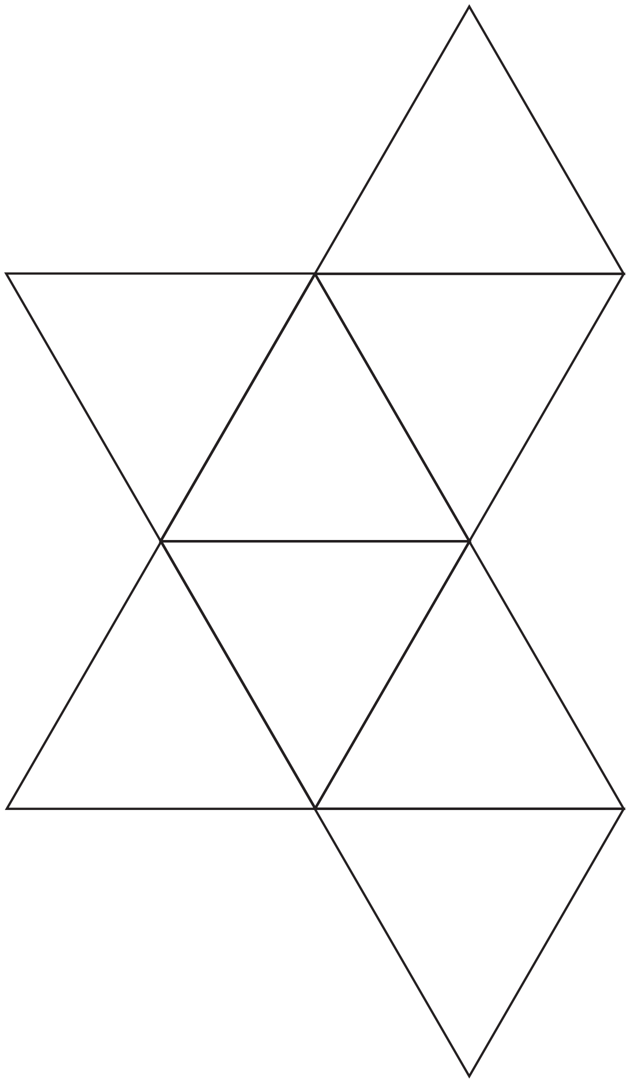
An *octahedron* is a solid with 8 faces.

One way to make an octahedron is to stick together two square-based pyramids.

1. Make an octahedron with all its edges the same length, from two identical square-based pyramids.

   Another method uses a net to make the solid.

2. Use the following net to make an octahedron:



ACTIVITY 6.5.1 *Klein Cube*

A *Klein Cube* is a three-dimensional version of a Mobius Strip (*August Mobius* was a pupil of the great mathematician *Carl Friedrich Gauss* (1777-1855)), and is named after its inventor, the German mathematician *Felix Klein* (1849-1928). He designed the *Klein Bottle* which is a three dimensional shape with only one surface; the model given here is based on the design of the Klein Bottle.

When you have constructed it, imagine that you start to paint the outside blue and continue painting along any joining surface. You will eventually find the whole shape (inside and outside) has been painted blue; hence it has only *one* surface.

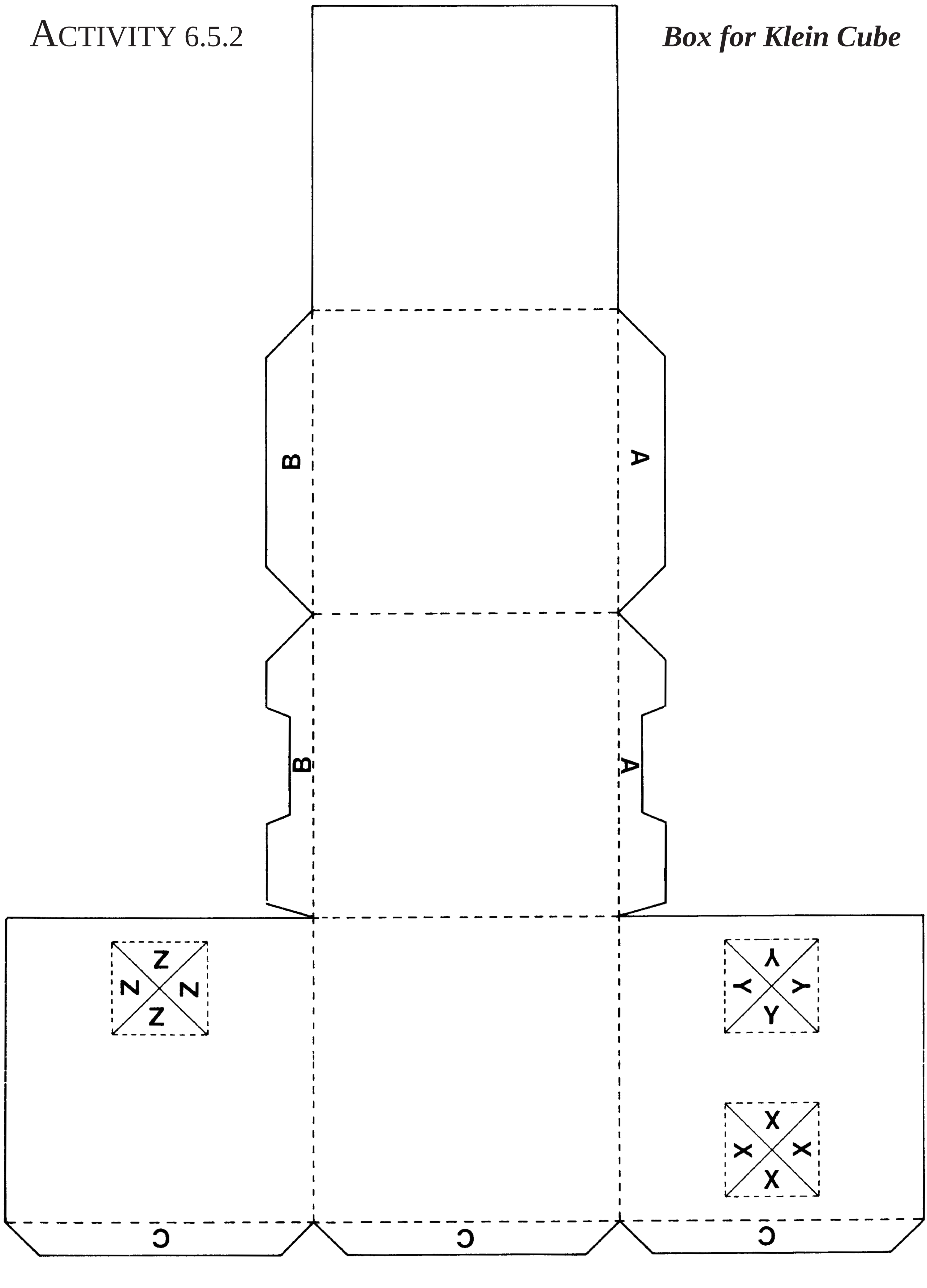
Instructions for making the Klein Cube

Activity pages 6.5.2 and 6.5.3

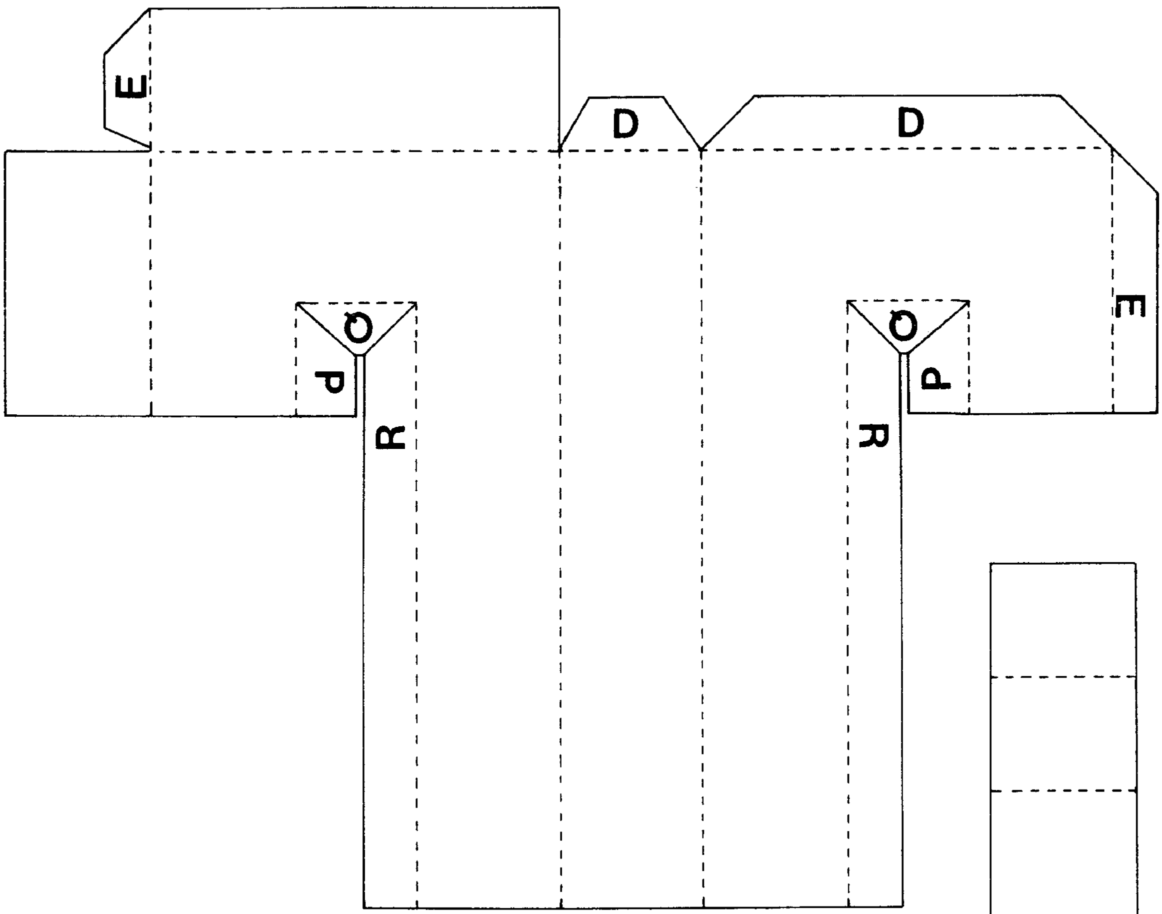
1. First cut out around the solid line of the edge of the net on page 6.5.2. Score and crease all the dotted lines to make tabs A, B and C.
2. Cut along solid lines in X, Y and Z and score along the dotted lines.
2. Fold up to make a box. Glue tabs A and B but NOT C.
3. Push tabs X and Y to outside of box and tab Z inside.
4. Next make the Tube for Klein Cube using the nets and instructions on page 6.5.3.
5. Now push the longer piece of the tube through the Y-opening of the box and into the Z-opening.
6. Glue the X, Y and Z tabs to the tube. The X and Y tabs go on the outside of the tube; the Z-tab goes inside the tube.
7. Glue tabs C to complete your Klein Cube.

ACTIVITY 6.5.2

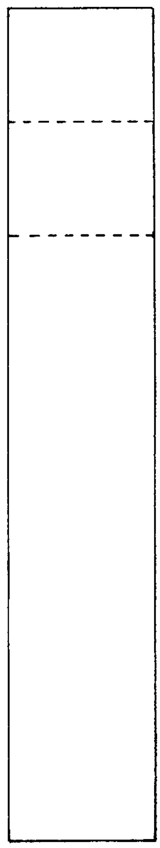
Box for Klein Cube



ACTIVITY 6.5.3 *Tube for Klein Cube*

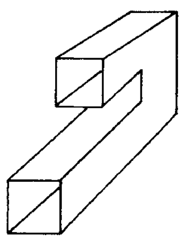


Tube for Klein Cube



Using the nets on this page,

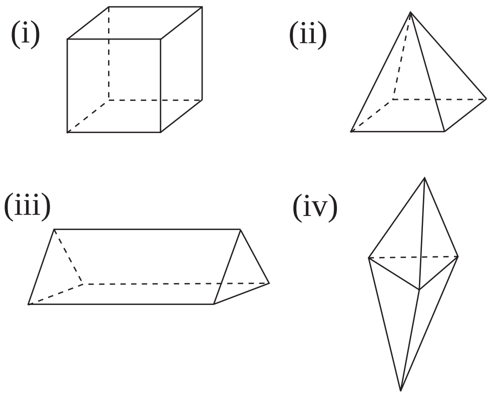
1. Cut out around the solid line of the edge of the larger net.
2. Cut along solid lines PQ and QR.
3. Score and crease all dotted lines to make tabs D, E, P, Q and R.
4. Fold up and glue tabs D and then tabs E.
5. Cut out, score and crease the smaller net.
6. Fold it and glue tabs P, Q and R to complete the tube.
   (It is easiest to do P and Q first, then R.)



Finished Tube

ACTIVITY 6.6 *Euler's Formula*

The particular result, named after its founder, the famous Swiss mathematician *Léonard Euler*, (1707-1783) is an example of a *topological invariant*; that is, something that remains constant for particular shapes.



1. For each of the shapes opposite, find
   (a) the number of edges, e
   (b) the number of vertices, v
   (c) the number of faces, f.

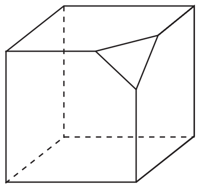
2. Show that

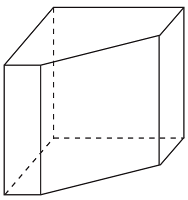
   $e + 2 = v + f$

   for each of these shapes.

Of course, verifying a formula for a few examples is no proof that the result is true for all such shapes. We will look at ways of trying to contradict the formula, as one method of establishing the generality of the result.

3. Suppose we cut off one corner of a cube.
   (a) How many more edges, vertices and faces are there?
   (b) Does the result still hold?



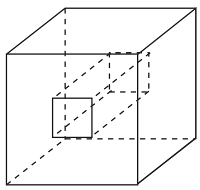


4. A slice is taken off a cube. Again, how many more edges, vertices and faces are there?

5. Try changing a cube in other ways and in each case check whether Euler's formula still holds.

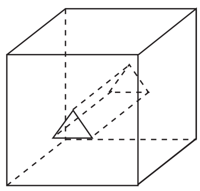
Extensions

1. Suppose that a square hole is made right through the cube. Does Euler's formula now hold?



2. Try making a triangular hole right through a cube. Does Euler's formula now hold?

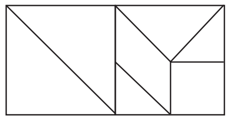
3. Put a similar hole in one of the other shapes used above. Does Euler's formula still hold?



ACTIVITIES 6.1 - 6.3 *Notes for Solutions*

Notes and solutions given only where appropriate.

6.1 Most of these questions should not prove difficult, except possibly the final one, in which you have to form a rectangle. The key to this is to form two squares and join them together as shown (not to scale) here.

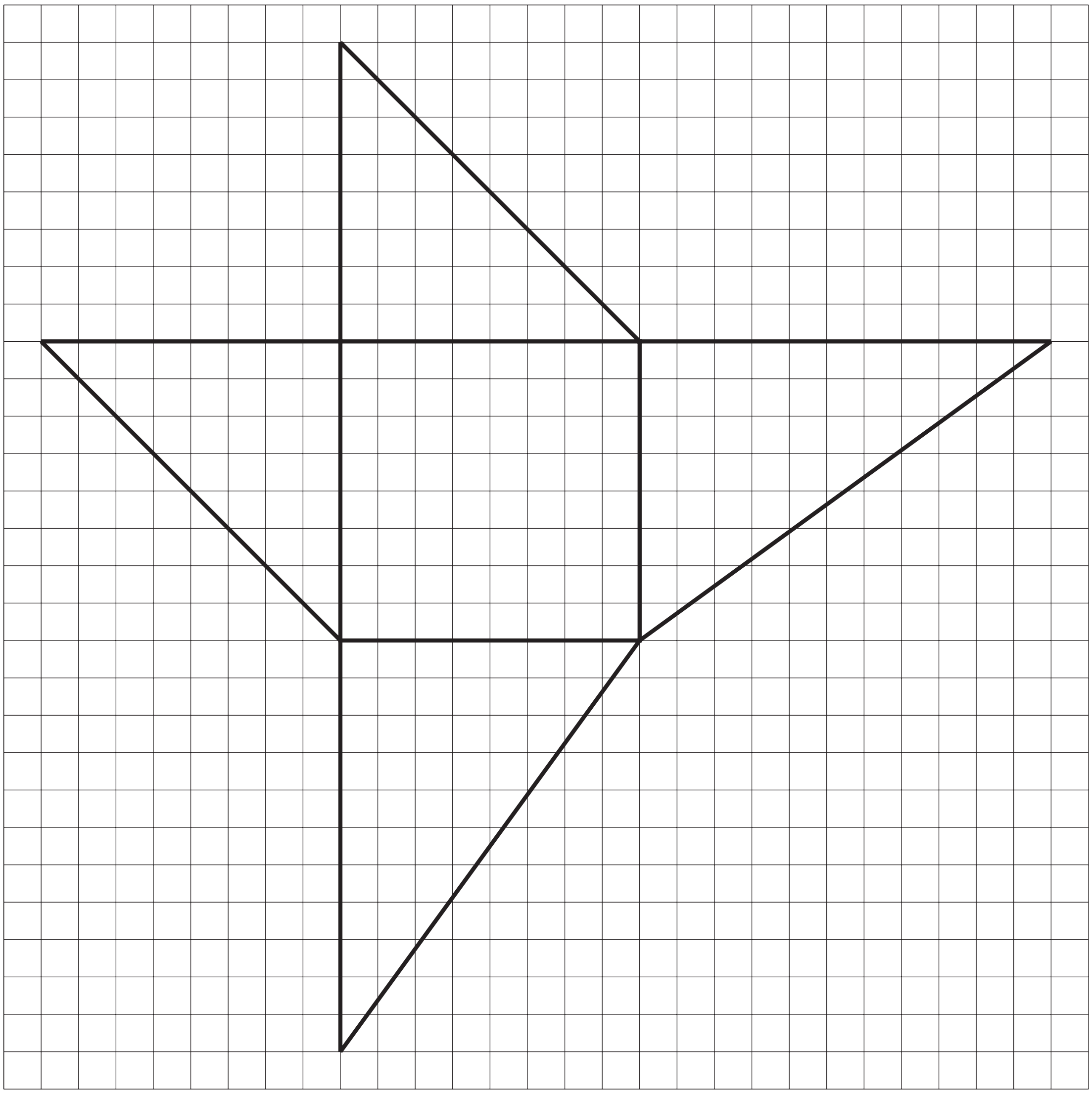


You can also make many other interesting shapes!

6.3 The best approach to drawing the nets is to draw accurate diagrams of the cross-sections of the pyramids. The alternative is to use Pythagoras' Theorem as developed in Unit 3.

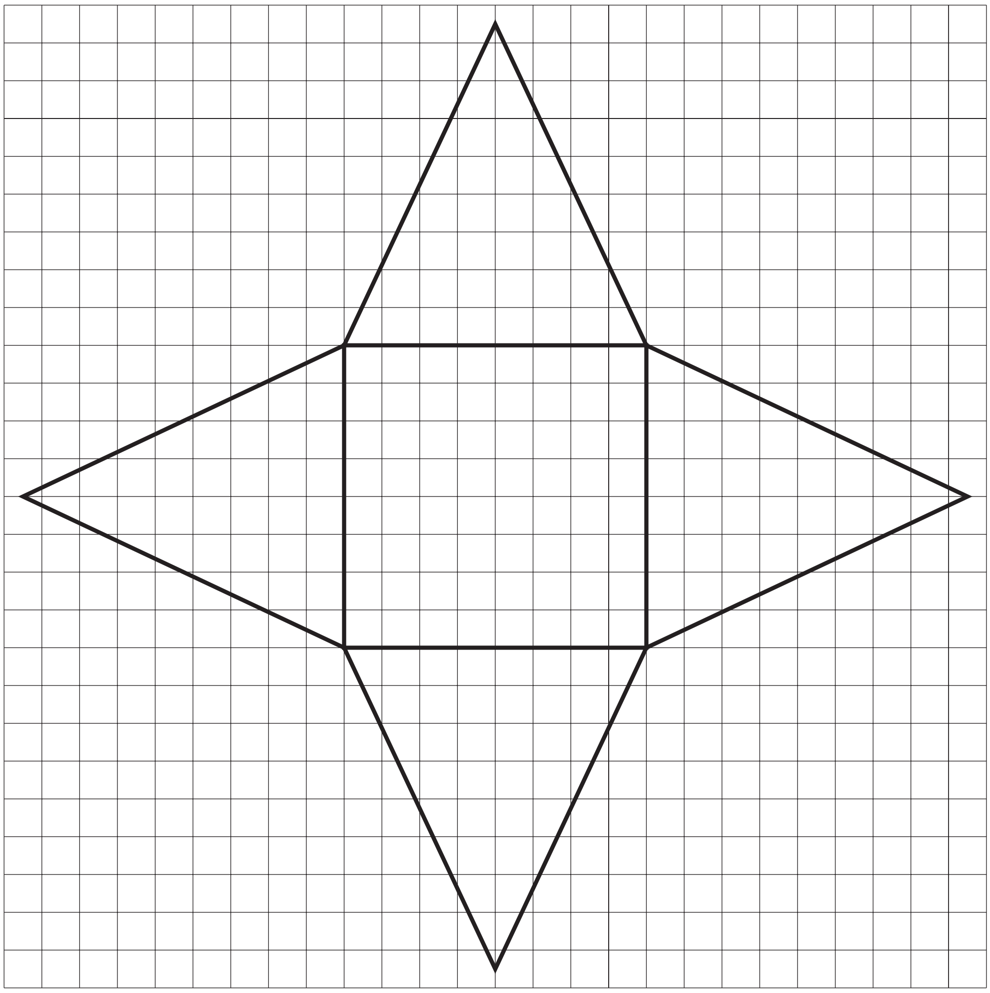
Possible nets for the 2 cm cube follow:

1.



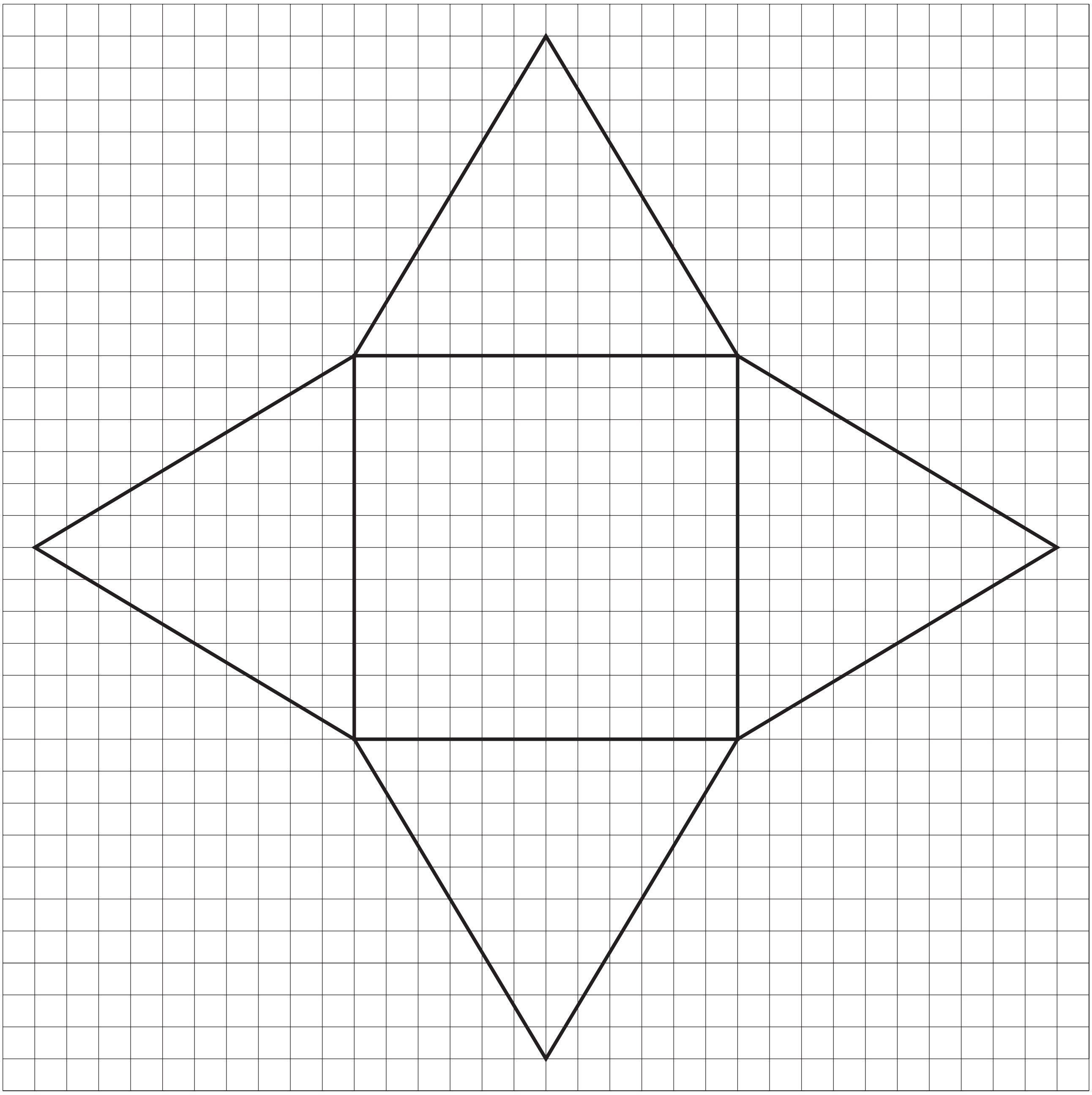
ACTIVITY 6.3 *Notes for Solutions*

6.3 1. (continued)



ACTIVITY 6.3 *Notes for Solutions*

6.3 2. Here is a possible net:



6.6 1. (i) $e = 12$, $v = 8$, $f = 6$ (ii) $e = 8$, $v = 5$, $f = 5$

(iii) $e = 9$, $v = 6$, $f = 5$ (iv) $e = 9$, $v = 5$, $f = 6$

3. 3, 2, 1; Yes 4. 3, 2, 1; Yes

Extensions 1. Yes 2. Yes 3. Yes