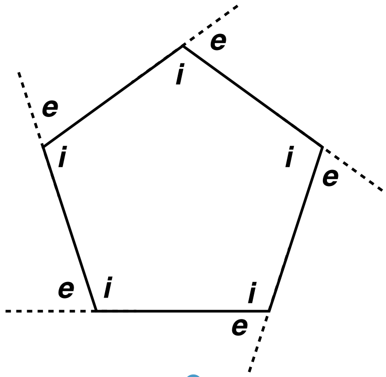
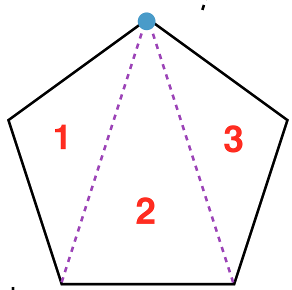
Angle Properties of Polygons

Starter

1. In the diagram, the angles marked e are the exterior angles. The angles marked i are the interior angles. where Write down a formula connecting exterior and interior angles, giving a reason for your answer.



2. Consider the pentagon. Starting from one **vertex** of the shape, we can draw **lines** to the the other vertices. For a pentagon this creates **3 triangles**.



Draw the following shapes and decide how many triangles each shape can be split into.

N.B. The **line(s)** you draw to split the shape into triangles cannot cross.

(a) Square (b) Hexagon (c) Octagon

Geogebra: Angle properties of polygons (click on "Inside△")

Working: (a) Square 2 triangles

3. Using your answers to 2, and using the what you know about the sum of the angles in a triangles, calculate the ***sum of the interior angles*** for:

(a) Square (b) Pentagon (c) Hexagon (d) Octagon

Working: (a) Square: 2 triangles so sum of interior angles $= 2 \times 180^o = 360^o$

Notes

A polygon is a closed $2-$dimensional shape with straight sides.
From the starter we found that for a polygon:

$$\text{Exterior angle} + \text{Interior angle} = 180^o$$

Geogebra: Angle properties of polygons

Sum of interior angles

Geogebra: Sum of interior angles of 3-6-sided polygons

From the starter, we found that (there is no need to copy the table):

Shape	Number of sides	Number of triangles	Sum of interior angles
Square	4	2	2 × 180º = 360º
Pentagon	5	3	3 × 180º = 540º
Hexagon	6	4	4 × 180º = 720º
Octagon	8	6	6 × 180º = 1080º

As can be seen, the number of triangles in a shape is 2 less than the number of its sides.
Therefore, for an $n-$sided polygon, there would be $(n-2)$ triangles.
Multiplying by 180^o gives:

Sum of interior angles of an $n-$sided polygon $= 180^o(n-2)$

Geogebra: Interior and exterior angles of a pentagon (interesting alternative "proof")

E.g.1 Find the sum of the interior angles of:
(a) a heptagon (7 sides) (b) a $12-$sided polygon (dodecagon)

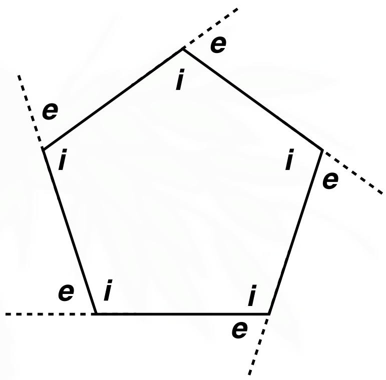
Working: (a) Sum of the interior angles of an $7-$sided polygon $= 180^o(7-2)$
$= 180^o \times 5$
$= 900^o$

Regular polygons
A ***regular polygon*** is a polygon whose ***sides*** are all the ***same length*** and whose ***angles*** are all ***equal***.

E.g. 2 What is the usual name for:
(a) a $3-$sided regular polygon (b) a $4-$sided regular polygon

Working: (a) Equilateral triangle

Sum of the exterior angles
Consider a regular pentagon i.e. a pentagon whose sides are all equal in length and whose angles are all equal.



Sum of the interior angles of a pentagon $= 540^o$.

Size of ***each interior angle*** of a regular pentagon $= \frac{540^o}{5} = 108^o$

Size of each exterior angle $= 180^o - 108^o = 72^o$

Sum of exterior angles $= 5 \times 72^o = 360^o$

In fact, this is true for all polygons:

Sum of exterior angles $= 360^o$

Size of one exterior angle in a regular polygon
The sum of the exterior angles of any polygon is 360^o.
For an $n-$sided polygon the exterior angles are all equal. Therefore:

Size of one exterior angle of an $n-$sided polygon $= \frac{360^o}{n}$

E.g. 3 Calculate the size of one ***exterior*** angle for a: polygon with:
(a) hexagon (b) a $16-$sided polygon

Working: (a) A hexagon has 6 sides
Exterior angle of a hexagon $= \frac{360^o}{n} = \frac{360^o}{6} = 60^o$

E.g. 4 A regular polygon has the following ***exterior*** angle. Calculate how many sides it has.

(a) 24^o (b) 40^o

Working: (a) **Exterior angle** $= \frac{360^o}{n}$**:**

$$24^o = \frac{360^o}{n}$$
$$n = \frac{360^o}{24^o}$$
$$n = 15$$

The regular polygon has 15 sides.

The equations **Exterior angle** $= \frac{360^o}{n}$ and **Exterior angle** $+$ **Interior angle** $= 180^o$ can be combined in questions.

E.g. 5 Calculate the size of one ***interior*** angle for a regular polygon with:

(a) 13 sides (b) 23 sides

Give your answers to 1 d.p.

Working: (a) Exterior angle $= \frac{360^o}{n} = \frac{360^o}{13} = 27.69^o$ (2 d.p.)

Interior angle $= 180^o - 27.69^o = 153.3^o$ (1 d.p.)

E.g. 6 A regular polygon has the following interior angle. Calculate how many sides it has.

(a) 144^o (b) 160^o

Working: (a) Exterior angle $= 180^o - 144^o = 36^o$

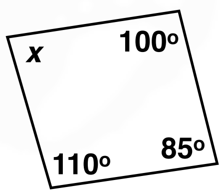
Exterior angle $= \frac{360^o}{n}$**:**

$$36^o = \frac{360^o}{n}$$
$$n = \frac{360^o}{36^o}$$
$$n = 10$$

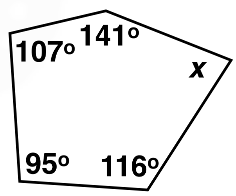
The regular polygon has 10 sides i.e. it is a decagon

E.g. 7* Find the value of x in these polygons

(a)



(b)



Working: (a) The polygon has 4 sides.

Sum of the interior angles $= 180^o(4 - 2) = 360^o$

$x = 360^o - 100^o - 85^o - 110^o = 65^o$

Videos: Angles in polygons

Solutions to Starter and E.g.s

Exercise

p58 Ex 15.2 Qu 1-7, 9

Summary

Exterior angle $+$ Interior angle $= 180^{o}$

Sum of interior angles of an $n-$sided polygon $= 180^{o}(n-2)$

Sum of exterior angles $= 360^{o}$

Exterior angle of an $n-$sided regular polygon $= \dfrac{360^{o}}{n}$

Textbook answers (only available during a lockdown)