Converting Recurring Decimals To Fractions

Starter

1. **(Review of last lesson)** Imani is a phenomenal rugby player in 8 games she scored 17 tries. Given that each try is worth 5 points, calculate her average points per game.

2. Write down the decimal equivalent of: (a) $\frac{1}{3}$ (b) $\frac{2}{3}$

Notes

$0.333...$ and $0.666...$ are examples of ***recurring decimals*** — they are when the digits are repeated continuously.

Notation

Dot notation is used to make the writing of recurring decimals quicker.

E.g. $2.555... = 2.\dot{5}$

Two dots can be used to show that more than one digit is being repeated>

E.g. $0.\dot{7}\dot{2} = 0.727272...$

When the two dots are separated, the digits in between the dots are also repeated.

E.g. $8.\dot{6}1\dot{4} = 8.614614614...$

E.g. $0.52\dot{3}189\dot{9} = 0.52318931893189...$

E.g. 1 Write the following recurring decimals in dot notation.

(a) $43.888...$ (b) $0.676767...$

(c) $9.463463463...$ (d) $0.7382382382...$

Working: (a) $43.888... = 43.\dot{8}$

E.g. 2 Convert these recurring decimals from dot notation to normal decimals.

(a) $0.\dot{2}$ (b) $4.3\dot{7}$ (c) $17.\dot{4}61\dot{8}$ (d) $0.93\dot{8}4615\dot{}$

Working: (a) $0.\dot{2} = 0.222...$

Converting recurring decimals to fractions

A recurring decimal is truncated when entered into a calculator so if a recurring decimal is used in a calculation it introduces a rounding error. To avoid this we can convert the recurring decimal to a fraction.

To convert a recurring decimal to a fraction, two recurring decimals with the same recurring part must be subtracted from one another.

E.g. 3 Express $0.\dot{4}$ as a fraction.

Working: $0.\dot{4} = 0.444...$

Let $x = 0.444...$

Multiply by 10***:*** $10x = 4.444...$

We now have two decimals with the same recurring part:

$$x = 0.444...$$
$$10x = 4.444...$$

When the first equation is subtracted from the second, the recurring part will disappear.

Subtracting: $10x - x = 4.444... - 0.444...$

$$9x = 4$$
$$x = \frac{4}{9}$$

E.g. 4 Express $0.\dot{7}$ as a fraction.

When there are ***two recurring digits***, multiplying by 10 won't be enough.

For example, let $x = 0.\dot{9}\dot{3} = 0.939393....$

Then $10x = 9.39393...$

Comparing the recurring parts we see that they are not the same:

$$x = 0.939393...$$
$$10x = 9.39393...$$

E.g. 5 Express $0.\dot{9}\dot{3}$ as a fraction in its lowest terms.

Working: $0.\dot{9}\dot{3} = 0.939393...$

Let $x = 0.939393...$

Multiply by 100***:*** $100x = 93.939393...$

Subtracting: $100x - x = 93.939393... - 0.939393...$

$$99x = 93$$
$$x = \frac{93}{99} = \frac{31}{33}$$

E.g. 6 Express these recurring decimals as fractions in their lowest terms.

(a) $0.\dot{1}\dot{6}$ (b) $0.\dot{7}\dot{2}$ (c) $0.4\dot{5}\dot{8}$

What happens when there is a non-recurring digit after the decimal point.

For example, $0.8\dot{5} = 0.8555...$ or $0.94\dot{6} = 0.94666...$

In such situation, we need to multiply by 10, 100 so that the ***recurring part is next to the decimal point***.

E.g. 7 Express $0.8\dot{5}$ as a fraction in its lowest terms.

Working: $0.8\dot{5} = 0.8555...$

Let $x = 0.8555...$

Multiply by 10 ***to get recurring part next to decimal point:***

$$10x = 8.555...$$

Multiply $x = 0.8555...$ ***by*** 100***:*** $100x = 85.555...$

Subtracting: $100x - 10x = 85.555... - 8.555...$

$$90x = 77$$

$$x = \frac{77}{90}$$

E.g. 8 Express these recurring decimals as fractions in their lowest terms.

(a) $0.3\dot{2}$ (b) $0.94\dot{6} = 0.94666...$ (c) $0.4\dot{9}\dot{5}$

Video: Recurring decimals to fractions

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p10 E1.1 Qu 1-2
A*-G class textbook:	p10 E1.1 Qu 1-2
9-1 homework book:	p4 E1.1 Qu 1-3
A*-G homework book:	p3 E1.1 Qu 1-3

Summary

Dot notation is used to make the writing of recurring decimals quicker.
Two dots can be used to show that more than one digit is being repeated.
When the two dots are separated, the digits in between the dots are also repeated.

Converting recurring decimals to fractions:
To convert a recurring decimal to a fraction, two recurring decimals with the same recurring part must be subtracted from one another.
When there is a non-recurring digit after the decimal point, multiply by 10, 100 so that the ***recurring part is next to the decimal point***.