

Formulae for Circumference and Area of a Circle

Starter

- (Review of last lesson)** Copy and complete these sentences.
 - The _____ is name for the perimeter of a circle.
 - A line connecting the centre of a circle to its perimeter is called a _____.
 - A _____ connects two points on the perimeter of a circle.
 - The longest line that can connect two points on the perimeter is called the _____.
 - The area of a circle between resembles a slice of pizza is called a _____.
 - The area between a chord and the perimeter of a circle is called a _____.

Notes

When the circumference is divided by the diameter we get a number that is a little over 3 in fact it is the number 3.14159... and it is given the letter π which is pronounced "pi".

There is no need to copy this.

The number, $\pi = 3.14159...$

The symbol π was introduced in 1706 by Welshman William Jones but was only fully adopted 50 years later by the great mathematician Leonard Euler.

π is an irrational number (proved in 1767 by the Swiss mathematician Johann Heinrich Lambert)

$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ (discovered in the 17th century by Gottfried Leibniz and John Gregory). An irrational number cannot be expressed as a single fraction.

The great Indian mathematician Ramanujan produced the formula

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26390n)}{(n!)^4 396^{4n}}$$

The two Ukrainians Gregory and David Chudnovsky derived this formula in the 1980s:

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} (-1)^n \times \frac{(6n)!}{(3n)!(n!)^3} \times \frac{163096908 + 6541681608n}{(262537412640768000)^{n+\frac{1}{2}}}$$

See Chapter 4 of Alex's Adventures in Numberland by Alex Bellos, which is in the school library, for more amazing number facts.

Please start copying again.

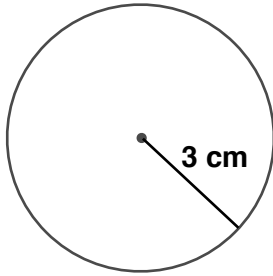
Formula for the circumference of a circle

Since $\pi = \frac{C}{d}$ we can rearrange to get $C = \pi d$ i.e. Circumference = $\pi \times$ diameter

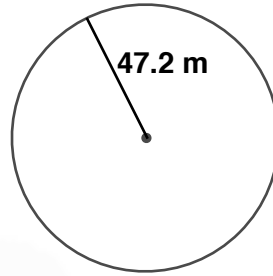
Since the diameter is twice the radius, we can replace d by $2r$: $C = 2\pi r$

E.g. 1 Calculate the circumference of these circles, giving your answers to 3 s.f..

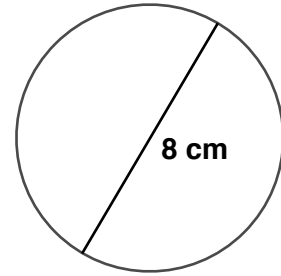
(a)



(b)



(c)



Working: (a) Circumference, $C = 2\pi r = 2 \times \pi \times 3 = 6\pi = 18.8 \text{ cm}$ (3 s.f.)

E.g. 2 The minute hand of a clock is 10 cm long. How far does the tip travel in 1 hour? Give your answer in terms of π .

E.g. 3 The circumference of a circle is 100 cm long. Calculate the radius of the circle, giving your answer to 3 s.f..

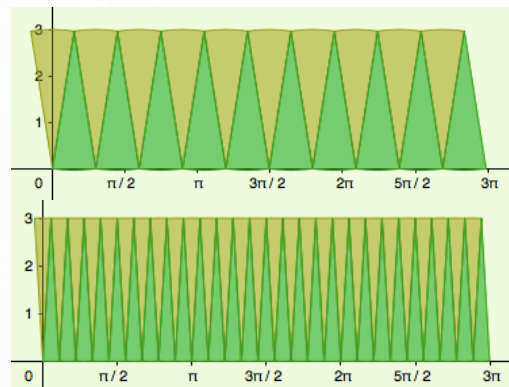
Formula for the area of a circle

Click on the geogebra link below to see how the area of a circle can be made up of sectors.

Geogebra: [Area of circle with sectors](#)

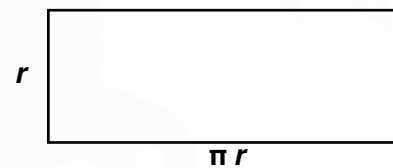
As the number of sectors increases, the shape starts to look like a parallelogram and then a rectangle.

The green and yellow diagrams are for a circle whose radius is 3 and whose circumference is 6π .



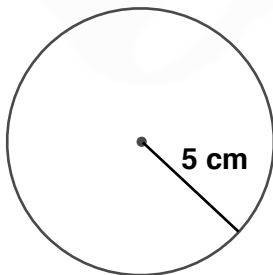
For a circle of radius r , the rectangle that the sectors approximate has width r and length πr . So the formula for the area of a circle is:

Area of circle, $A = \pi r^2$

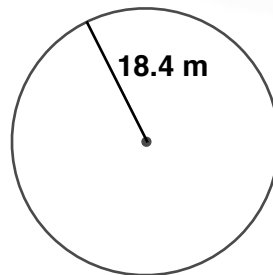


E.g. 4 Calculate the area of these circles, giving your answers to 3 s.f..

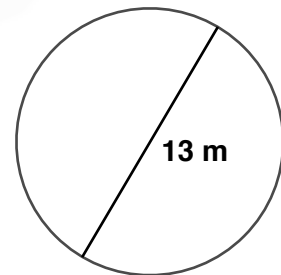
(a)



(b)



(c)



Working: (a) Area, $A = \pi r^2 = \pi \times 5^2 = 25\pi = 78.5 \text{ cm}$ (3 s.f.)

E.g. 5 A circle has area 160 cm^2 . Find its diameter to 3 s.f..

Video: [Circumference of a circle](#)

Video: [Area of a circle](#)

Video: [Perimeter of a semi-circle](#)

[Solutions to Starter and E.g.s](#)

Exercise

p79 Ex 16.4 Qu 1-11

Summary

Circumference: $C = 2\pi r$ or $C = \pi d$

Area of a circle: $A = \pi r^2$

[Textbook answers \(only available during a lockdown\)](#)

