HCF/LCM

Starter

1. **(Review of last lesson)**
   Express 360 as the product of prime factors. Give your answer in index notation.

2. Consider the numbers 8 and 20.
   (a) Find the largest factor that is common to both numbers.
   (b) Find the smallest number that both numbers will go into without leaving a remainder.

Notes

The answer to 2(a) in the starter is called the ***highest common factor***.
The answer to 2(b) in the starter is called the ***lowest common multiple***.

Highest Common Factor (HCF) — the highest number which divides exactly into two or more given numbers.

Lowest Common Multiple (LCM) — the smallest number which each of the two or more given numbers will divide into without leaving a remainder.

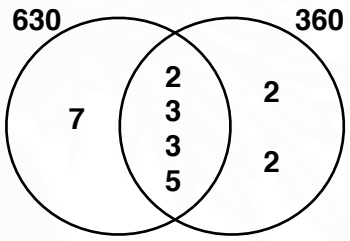
Using a Venn diagram to find the HCF & LCM

While the method of writing all the factors and multiples of the numbers down can work, it can be more difficult when the two numbers are large. In such situations, use a factor tree to find the prime factors and then a Venn diagram to find the HCF and LCM.

From before, we have

$$630 = 2 \times 3^2 \times 5 \times 7$$
$$360 = 2^3 \times 3^2 \times 5.$$

Prime factors that are common go in the ***intersection*** (or overlap) so the Venn diagram would look like this.



The **HCF** is the product of the prime factors in the **intersection**.

$$\text{HCF} = 2 \times 3 \times 3 \times 5 = 90$$

The **LCM** is the product of **all** the prime factors in the **whole diagram**.

$$\text{LCM} = 7 \times 2 \times 3 \times 3 \times 5 \times 2 \times 2 = 2520$$

Remember: the **HCF** must be equal to or **smaller than** the **smallest number**.
the **LCM** must be equal to or **larger than** the **largest number**.

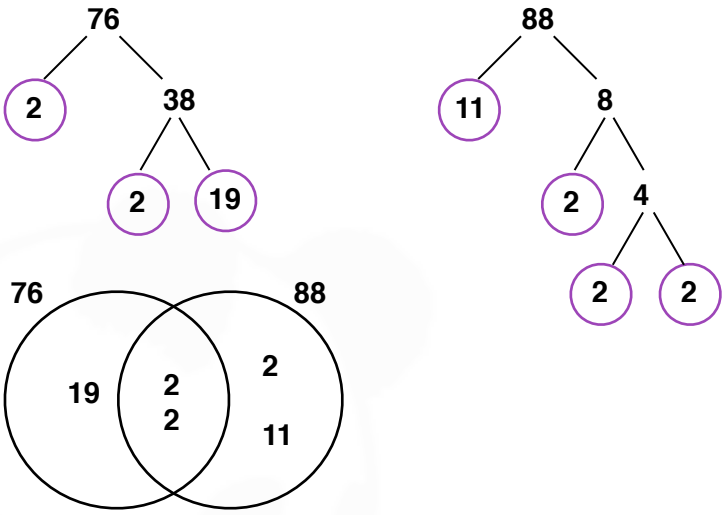
Success Criteria — finding the HCF and LCM

1. Draw factor tree for each number in order to get their prime factor.
2. Draw a Venn diagram, with one circle for each number.
3. Enter the prime factors in the Venn diagram, making sure that common factors go in the intersection (or overlap).
4. The **HCF** is the product of the prime factors in the **intersection**.
   The **LCM** is the product of **all** the prime factors in the **whole diagram**.

E.g. 1 Find the highest common factor and lowest common multiple of these numbers.

(a) 76 and 88 (b) 450 and 660

Working: (a)



HCF $= 2 \times 2 = 4$ ***HCF = product of intersection***

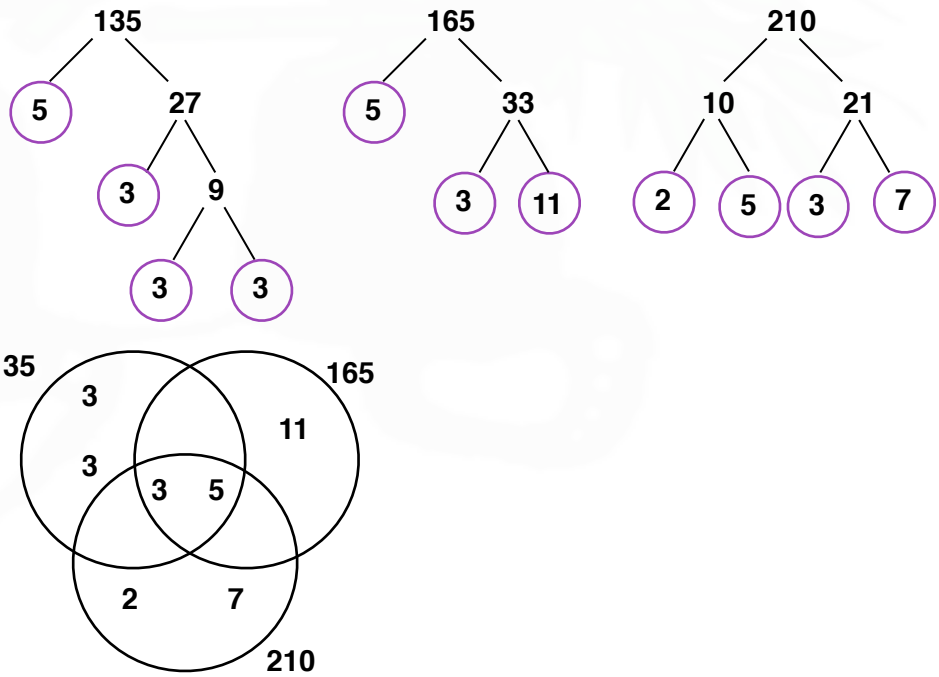
LCM $= 19 \times 2 \times 2 \times 1 \times 11 = 1672$ ***LCM = whole diagram***

When there are three numbers , the Venn Diagram has three circles.

E.g. 2 Find the HCF and LCM of the numbers

(a) 135, 165 and 210 (b) 102, 612 and 6545

Working: (a)



HCF is $3 \times 5 = 15$

LCM is $3 \times 3 \times 3 \times 5 \times 2 \times 7 \times 11 = 20790$

E.g. 3 Jess swims once every 21 days. Peter swims once every 35 days. They both went swimming today. How many days will it be before they swim on the same day again?

Product of the HCF & LCM

For the numbers 630 and 360, the HCF is 90 and the LCM is 2520.

E.g. 4 (a) Find the product of 630 and 360

(b) Find the product of their HCF and LCM. What do you notice?

The product of two numbers is equal to the product of their HCF and LCM. The same is true for three numbers etc.

E.g. 5 The HCF and LCM of two numbers is 70 and 186 respectively. Given that one of the numbers 210, find the other number.

Video: Finding HCF and LCM

Video: Finding the HCF and LCM using Venn diagrams 1

Video: Finding the HCF and LCM using Venn diagrams 2

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p127 M5.2 Qu 1-6
A*-G class textbook:	p119 M5.2 Qu 5-12
9-1 homework book:	p44 M5.1/5.2 Qu 4-12
A*-G homework book:	p32 M5.2 Qu 4-10

Summary

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