# Independent Events and Tree Diagrams

**Starter**

1. **(Review of last lesson)**
Stella and Paul each have three cards numbered $1$, $2$ and $3$. They each take one of the other's cards and add the total. What is the probability the total will be an odd number?
***Hint:*** list the possible outcomes in a probability space diagram.

2. **(Review of previous material)**
Without using a calculator, find: (a) $\frac{6}{11} \times \frac{5}{11}$ (b) $\frac{4}{9} \times \frac{3}{5}$

3. **(Review of previous material)**
Without using a calculator, find: (a) $0.6 \times 0.4$ (b) $0.7 \times 0.1$

**Notes**

Successive events are when we have one event followed by another (e.g. rolling a dice and flipping a coin). In the previous lesson we looked at two events whose outcomes all had the same probability (e.g. flipping two coins). What happens when the outcomes do not have the same probability?

***Independent events***

Events are independent when the outcome of one event does not depend on the outcome of the other i.e. ***one event does not affect the outcomes of another event***.

***E.g. 1*** Decide whether the following events are independent or not.
(a) Flipping two coins.
(b) Penalties in a penalty shoot-out.
(c) Rolling two coins.
(d) The weather on two consecutive days.
(e) The weather on days separated by 6 months.

***Working:*** (a) Independent.

In general, events are independent when they are ***physically independent*** from each other.

If events $A$ and $B$ are independent, the probability of event $A$ happening followed by event $B$ is $P(A) \times P(B)$

$$P(A,\, B) = P(A) \times P(B)$$

***E.g. 2*** The probability that Irina is late for school is $0.3$; the probability that Helena is late is $0.2$. What is the probability that both girls are late for school?

***Working:*** $P(\text{late, late}) = P(\text{late}) \times P(\text{late}) = 0.3 \times 0.2 = 0.06$

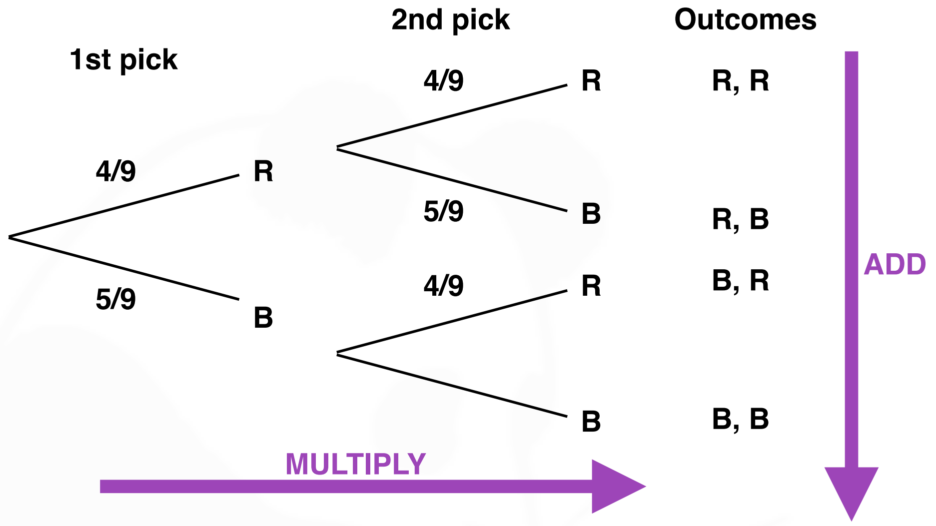
***E.g. 3*** A coin is flipped $3$ times. What is the probability of getting $3$ heads?

***E.g. 4*** A dice is rolled twice. Find the probability of getting a two on just one of these rolls.

***Tree diagrams***

A tree diagram is a useful tool to solve probability problems and would make it easier to do E.g. 4.

Imagine a bag has $9$ discs — $4$ red and $5$ blue. A disc is chosen at random, its colour noted and then replaced in the bag. Another disc is then chosen.




***Key ideas***

- Probabilities go on the branches and outcomes at the end of each branch.
- The sum of the probabilities on the branches must be $1$.
- When going across the tree diagram, we multiply probabilities.

  ***E.g.*** $P(\text{2 red discs}) = P(R, R)$

  $$= P(R) \times P(R)$$
  $$= \frac{4}{9} \times \frac{4}{9}$$
  $$= \frac{16}{81}$$

- When going down the tree diagram, we add the probabilities.

  ***E.g.*** $P(\text{same colour}) = P(R, R) + P(B, B)$

  $$= P(R) \times P(R) + P(B) \times P(B)$$
  $$= \frac{4}{9} \times \frac{4}{9} + \frac{5}{9} \times \frac{5}{9}$$
  $$= \frac{16}{81} + \frac{25}{81}$$
  $$= \frac{41}{81}$$

***Two common questions***

1. What is the probability of getting ***only one*** red disc?

***Working:*** To get one red disc, we could choose a red disc first and then a blue disc ***or*** could a blue one and then a red one.

$$\begin{aligned} P(\text{only 1 red disc}) &= P(R,B) + P(R,B) \\ &= P(R)\times P(B) + P(B)\times P(R) \\ &= \frac{4}{9}\times\frac{5}{9} + \frac{5}{9}\times\frac{4}{9} \\ &= \frac{20}{81} + \frac{20}{81} \\ &= \frac{40}{81} \end{aligned}$$

***N.B.*** This is the same as finding the probability of getting different colours.

2. What is the probability of getting ***at least one*** blue disc?

***Working:*** "At least one blue disc" is the complementary event of "no blue discs".
"No blue discs" means that two red discs have been chosen

$$\begin{aligned} P(\text{at least 1 blue disc}) &= 1 - P(\text{no blue discs}) \\ &= 1 - P(\text{2 red discs}) \\ &= 1 - P(\text{R, R}) \\ &= 1 - \frac{4}{9}\times\frac{4}{9} \\ &= 1 - \frac{16}{81} \\ &= \frac{65}{81} \end{aligned}$$

***N.B.*** You could also do $P(\text{at least 1 blue disc}) = P(\text{1 blue disc}) + P(\text{2 blue discs})$

***Notation***

$R'$ is used to denote "not R" and is called ***complement*** notation.

For example, when rolling a dice $P(6) = \frac{1}{6}$ and $P(6') = \frac{5}{6}$

***E.g. 5*** While driving to work, Sarah passes through two sets of traffic lights. The probability the first set is green is $0.4$ and the probability the second is green is $0.3$.

(a) Draw a tree diagram.

(b) Find the probability that:

(i) neither set of lights is green

(ii) only one set of traffic lights is green

(iii) at least one set of traffic lights is green

***Hint:*** In your tree diagram use G for "green" and G' for "not green".

Video: Independent events
Video: Tree diagrams

Solutions to Starter and E.g.s

**Exercise**

p181 Ex 10.4 Qu 2, 3, 5, 7-15

**Summary**

***Successive events*** are when we have one event followed by another.

***Independent events*** are successive events where ***one event does not affect the outcome of another event***.

In general, events are independent when they are ***physically independent*** from each other.

If events $A$ and $B$ are independent: $P(A, B) = P(A) \times P(B)$

Tree diagrams

- The sum of the probabilities on the branches must be $1$.
- When going across the tree diagram, we multiply probabilities.
- When going down the tree diagram, we add the probabilities.

Textbook answers (only available during a lockdown)