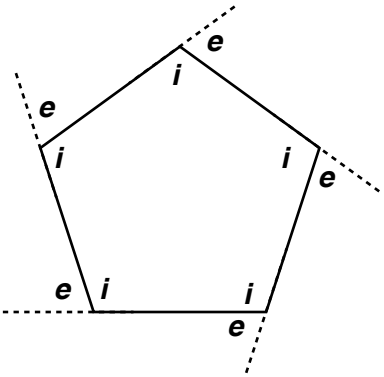
Angle Properties of Polygons

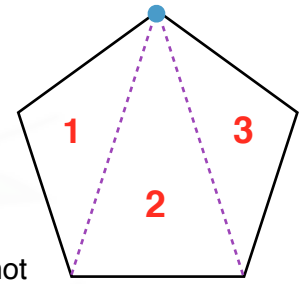
Starter

1. In the diagram, the angles marked e are the exterior angles. The angles marked i are the interior angles. where
Write down a formula connecting exterior and interior angles, giving a reason for your answer.



Working: Exterior angle + Interior angle $= 180^o$
Reason: angles on a straight line add up to 180^o

2. Consider the pentagon. Starting from one **vertex** of the shape, we can draw **lines** to the the other vertices.
For a pentagon this creates **3 triangles**.



Draw the following shapes and decide how many triangles each shape can be split into.

N.B. The **line(s)** you draw to split the shape into triangles cannot cross.

(a) Square (b) Hexagon (c) Octagon

Geogebra: **Angle properties of polygons** (click on "Inside△")

Working:
(a) Square 2 triangles
(b) Hexagon 4 triangles
(c) Octagon 6 triangles

3. Using your answers to 2, and using the what you know about the sum of the angles in a triangles, calculate the ***sum of the interior angles*** for:

(a) Square (b) Pentagon (c) Hexagon (d) Octagon

Working:
(a) Square: 2 triangles so sum of interior angles $= 2 \times 180^o = 360^o$
(b) Pentagon: 3 triangles so sum of interior angles $= 3 \times 180^o = 540^o$
(c) Hexagon: 4 triangles so sum of interior angles $= 4 \times 180^o = 720^o$
(d) Octagon: 6 triangles so sum of interior angles $= 6 \times 180^o = 1080^o$

E.g.1 Find the sum of the interior angles of:
(a) a heptagon (7 sides) (b) a $12-$sided polygon (dodecagon)

Working: (a) Sum of the interior angles of an $7-$sided polygon $= 180^o(7-2)$
$= 180^o \times 5$
$= 900^o$

(b) Sum of the interior angles of an $12-$sided polygon $= 180^o(12-2)$
$= 180^o \times 10$
$= 1800^o$

E.g. 2 What is the usual name for:
(a) a $3-$sided regular polygon (b) a $4-$sided regular polygon

Working: (a) Equilateral triangle

(b) Square

E.g. 3 Calculate the size of one ***exterior*** angle for a: polygon with:
(a) hexagon (b) a $16-$sided polygon

Working: (a) A hexagon has 6 sides

Exterior angle of a hexagon $= \dfrac{360^o}{n} = \dfrac{360^o}{6} = 60^o$

(b) Exterior angle of a $16-$sided polygon $= \dfrac{360^o}{n} = \dfrac{360^o}{16} = 22.5^o$

E.g. 4 A regular polygon has the following ***exterior*** angle. Calculate how many sides it has.
(a) 24^o (b) 40^o

Working: (a) **Exterior angle** $= \dfrac{360^o}{n}$**:** $24^o = \dfrac{360^o}{n}$

$n = \dfrac{360^o}{24^o}$

$n = 15$

The regular polygon has 15 sides.

(b) **Exterior angle** $= \dfrac{360^o}{n}$**:** $40^o = \dfrac{360^o}{n}$

$n = \dfrac{360^o}{40^o}$

$n = 9$

The regular polygon has 9 sides. i.e. it is a nonagon.

E.g. 5 Calculate the size of one ***interior*** angle for a regular polygon with:
(a) 13 sides (b) 23 sides
Give your answers to 1 d.p.

Working: (a) Exterior angle $= \dfrac{360^o}{n} = \dfrac{360^o}{13} = 27.69^o$ (2 d.p.)
Interior angle $= 180^o - 27.69^o = 153.3^o$ (1 d.p.)

(b) Exterior angle $= \dfrac{360^o}{n} = \dfrac{360^o}{23} = 15.65^o$ (2 d.p.)
Interior angle $= 180^o - 15.65^o = 164.3^o$ (1 d.p.)

E.g. 6 A regular polygon has the following interior angle. Calculate how many sides it has.
(a) 144^o (b) 160^o

Working: (a) Exterior angle $= 180^o - 144^o = 36^o$

Exterior angle $= \dfrac{360^o}{n}$: $36^o = \dfrac{360^o}{n}$

$n = \dfrac{360^o}{36^o}$

$n = 10$

The regular polygon has 10 sides i.e. it is a decagon

(b) Exterior angle $= 180^o - 160^o = 20^o$

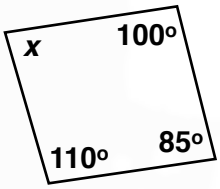
Exterior angle $= \dfrac{360^o}{n}$: $20^o = \dfrac{360^o}{n}$

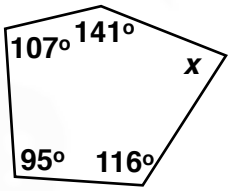
$n = \dfrac{360^o}{20^o}$

$n = 18$

The regular polygon has 18 sides.

E.g. 7* Find the value of x in these polygons
(a) (b)





Working: (a) The polygon has 4 sides.
Sum of the interior angles $= 180^o(4-2) = 360^o$
$x = 360^o - 100^o - 85^o - 110^o = 65^o$

(b) The polygon has 5 sides.
Sum of the interior angles $= 180^o(5-2) = 540^o$
$x = 540^o - 107^o - 141^o - 116^o - 95^o = 81^o$

Videos: Angles in polygons

Solutions to Starter and E.g.s

Exercise
p58 Ex 15.2 Qu 1-7, 9

Textbook answers (only available during a lockdown)