Changing the Subject of a Formula

Starter

1. **(Review of last lesson)**

Solve: (a) $2(3x-4)=35$ (b) $5(p+3)-7(p-4)=53$

(c) $\dfrac{4x+1}{5}-3=-10$ (d) $\dfrac{15}{x+4}=\dfrac{19}{x+3}$

Working: (a)

	$2(3x-4)=35$
Expand the brackets	$6x-8=35$
Add 8 ***to both sides***	$6x=43$
Divide both sides by 6	$x=\dfrac{43}{6}$

(b)

	$5(p+3)-7(p-4)=53$
Expand the brackets	$5p+15-7p+28=53$
Collect like terms	$-2p+43=53$
Subtract 43 ***from both sides***	$-2p=10$
Divide both sides by -2	$p=-5$

(c) ***The*** $4x+1$ ***can be considered to be in brackets***

	$\dfrac{(4x+1)}{5}-3=-10$
Add 3 ***to both sides***	$\dfrac{(4x+1)}{5}=-7$
Multiply both sides by 5	$4x+1=-35$
Subtract 1 ***from both sides***	$4x=-36$
Divide both sides by 4	$x=-9$

(d)

	$\dfrac{15}{x+4}=\dfrac{19}{x+3}$
Cross-multiply	$15(x+4)=19(x+3)$
Expand the brackets	$15x+60=19x+57$
Subtract $15x$ ***from both sides***	$60=4x+57$
Subtract 57 ***from both sides***	$3=4x$
Divide both sides by 4	$\dfrac{3}{4}=x$
Swap to get x ***on the LHS***	$x=\dfrac{3}{4}$

2. **(Review of previous material)**

The length of a rectangle is three times its width. Its perimeter is 32 m. Find its area.

Working: Let w be the width of the rectangle.

So the length is $3w$

Perimeter of a rectangle	$2(3w+w)=32$
Collect like terms in the bracket	$2\times 4w=32$
	$8w=32$
	$w=4$

So the length $=3\times 4=12$

$\therefore$ area $=12\times 4=48$ m^2.

3. What is the difference between an equation and a formula?

Working: Equation — one unknown and some numbers. See examples above.

Formula — a relationship connecting two or more quantities (at least two letters and some numbers). ***E.g.*** $C = 3m + 10$, $v = u + at$, $V = IR$

E.g. 1 Rearrange the formula to make x the subject:

(a) $3x - p = q$ (b) $y = mx + c$ (c) $s - t = t + kx$

Working: (a) ***x is positive so we don't want to move it to the other side***

Subtraction before multiplication $3x - p = q$

Add q to both sides $3x = q + p$

Divide both sides by 3 $x = \frac{q+p}{3}$

(b) ***x is positive so we don't want to move it to the other side***

Addition before multiplication $y = mx + c$

Subtract c from both sides $y - c = mx$

Divide both sides by m $\frac{y-c}{m} = x$

Swap the formula over $x = \frac{y-c}{m}$

(c) ***x is positive so we don't want to move it to the other side***

Addition before multiplication $s - t = t + kx$

Subtract t from both sides $s - 2t = kx$

Divide both sides by k $\frac{s-2t}{k} = x$

$x = \frac{s-2t}{k}$

E.g. 2 Rearrange the formula to make x the subject:

(a) $\frac{x}{c} = a + b$ (b) $\frac{x}{s} + 1 = t$ (c) $a = \frac{x}{m} + n$

Working: (a) ***x is positive so we don't want to move it to the other side***

$\frac{x}{c} = a + b$

Multiply both sides by c $x = c(a + b)$

(b) ***x is positive so we don't want to move it to the other side***

Addition before division $\frac{x}{s} + 1 = t$

Subtract 1 from both sides $\frac{x}{s} = t - 1$

Multiply both sides by s $x = s(t - 1)$

(c) *x is positive so we don't want to move it to the other side*

Addition before division $a = \frac{x}{m} + n$

Subtract n from both sides $a - n = \frac{x}{m}$

Multiply both sides by m $m(a - n) = x$

Swap the formula over $x = m(a - n)$

N.B. *Expand* brackets *before rearranging* ***unless the new subject is in front of the bracket***

E.g. 3 Rearrange the formula to make x the subject of the formula:

(a) $s(x + a) = b$ (b) $z = p(3x - y)$ (c) $x(c + d) = e$

Working: (a) *x is positive so we don't want to move it to the other side*

$s(x + a) = b$

x is not in front of the brackets so expand $sx + as = b$

Addition before multiplication

Subtract as from both sides $sx = b - as$

Divide both sides by s $x = \frac{b - as}{s}$

(b) *x is positive so we don't want to move it to the other side*

$z = p(3x - y)$

x is not in front of the brackets so expand $z = 3px - py$

Subtraction before multiplication

Add py to both sides $z + py = 3px$

Divide both sides by $3p$ $x = \frac{z + py}{3p}$

(c) *x is positive so we don't want to move it to the other side*

x is in front of brackets so do not expand $x(c + d) = e$

Divide both sides by $c + d$ $x = \frac{e}{c + d}$

N.B. When the new subject is negative, it is usually a good idea to *prioritise making it positive*

E.g. 3 Rearrange the formula to make y the subject of the formula:

(a) $e = u - gy$ (b) $w^2 = u^2 - ay$ (c) $t - 5y = b$

Working: (a) *y is negative so we need to move it to the other side*

$e = u - gy$

Add gy to both sides $e + gy = u$

Addition before multiplication

Subtract e from both sides $gy = u - e$

Divide both sides by g $y = \frac{u - e}{g}$

(b) *y is negative so we need to move it to the other side*

$$w^2 = u^2 - ay$$

Add ay to both sides

$$w^2 + ay = u^2$$

Addition before multiplication

Subtract w^2 from both sides

$$ay = u^2 - w^2$$

Divide both sides by a

$$y = \frac{u^2 - w^2}{a}$$

(c) *y is negative so we need to move it to the other side*

$$t - 5y = b$$

Add $5y$ to both sides

$$t = b + 5y$$

Addition before multiplication

Subtract b from both sides

$$t - b = 5y$$

Divide both sides by 5

$$\frac{t - b}{5} = y$$

Swap the formula over

$$y = \frac{t - b}{5}$$

Video: Changing the subject of a formula

Solutions to Starter and E.g.s

Exercise
p23 Ex 12.6 Qu 1ace…, 2-10

Textbook answers (only available during a lockdown)