

## Converting Recurring Decimals To Fractions

### Starter

1. **(Review of last lesson)** Imani is a phenomenal rugby player in 8 games she scored 17 tries. Given that each try is worth 5 points, calculate her average points per game.

**Working:** Total points =  $17 \times 5 = 85$   
Points per game =  $\frac{85}{8} = 10.625$

2. Write down the decimal equivalent of: (a)  $\frac{1}{3}$  (b)  $\frac{2}{3}$

**Working:** (a)  $\frac{1}{3} = 0.333\dots$  or  $0.\dot{3}$

(b)  $\frac{2}{3} = 0.666\dots$  or  $0.\dot{6}$

**E.g. 1** Write the following recurring decimals in dot notation.

- (a) 43.888... (b) 0.676767...  
(c) 9.463463463... (d) 0.7382382382...

**Working:** (a)  $43.888\dots = 43.\dot{8}$   
(b)  $0.676767\dots = 0.\dot{6}\dot{7}$   
(c)  $9.463463463\dots = 9.\dot{4}\dot{6}\dot{3}$   
(d)  $0.7382382382\dots = 0.\dot{7}\dot{3}\dot{8}\dot{2}$

**E.g. 2** Convert these recurring decimals from dot notation to normal decimals.

- (a)  $0.\dot{2}$  (b)  $4.\dot{3}\dot{7}$  (c)  $17.\dot{4}\dot{6}\dot{1}\dot{8}$  (d)  $0.93\dot{8}461\dot{5}$

**Working:** (a)  $0.\dot{2} = 0.222\dots$   
(b)  $4.\dot{3}\dot{7} = 4.373737\dots$   
(c)  $17.\dot{4}\dot{6}\dot{1}\dot{8} = 17.461846184618\dots$   
(d)  $0.93\dot{8}461\dot{5} = 0.93846158461584615\dots$

**E.g. 3** Express  $0.\dot{4}$  as a fraction.

**Working:**  $0.\dot{4} = 0.444\dots$   
Let  $x = 0.444\dots$   
**Multiply by 10:**  $10x = 4.444\dots$   
**We now have two decimals with the same recurring part:**  
 $x = 0.444\dots$   
 $10x = 4.444\dots$   
**When the first equation is subtracted from the second, the recurring part will disappear.**  
**Subtracting:**  $10x - x = 4.444\dots - 0.444\dots$   
 $9x = 4$   
 $x = \frac{4}{9}$

**E.g. 4** Express  $0.\dot{7}$  as a fraction.

**Working:**  $0.\dot{7} = 0.777\dots$   
Let  $x = 0.777\dots$   
**Multiply by 10:**  $10x = 7.777\dots$   
**Subtracting:**  $10x - x = 7.777\dots - 0.777\dots$   
 $9x = 7$   
 $x = \frac{7}{9}$

**E.g. 5** Express  $0.\dot{9}\dot{3}$  as a fraction in its lowest terms.

**Working:**  $0.\dot{9}\dot{3} = 0.939393\dots$   
Let  $x = 0.939393\dots$   
**Multiply by 100:**  $100x = 93.939393\dots$   
**Subtracting:**  $100x - x = 93.939393\dots - 0.939393\dots$   
 $99x = 93$   
 $x = \frac{93}{99} = \frac{31}{33}$

**E.g. 6** Express these recurring decimals as fractions in their lowest terms.

- (a)  $0.\dot{1}\dot{6}$                       (b)  $0.\dot{7}\dot{2}$                       (c)  $0.\dot{4}5\dot{8}$

**Working:** (a)  $0.\dot{1}\dot{6} = 0.161616\dots$   
Let  $x = 0.161616\dots$   
**Multiply by 100:**  $100x = 16.161616\dots$   
**Subtracting:**  $100x - x = 16.161616\dots - 0.161616\dots$   
 $99x = 16$   
 $x = \frac{16}{99}$

(b)  $0.\dot{7}\dot{2} = 0.727272\dots$   
 Let  $x = 0.727272\dots$   
**Multiply by 100:**  $100x = 72.727272\dots$   
**Subtracting:**  $100x - x = 72.727272\dots - 0.727272\dots$   
 $99x = 72$   
 $x = \frac{72}{99} = \frac{8}{11}$

(c)  $0.\dot{4}5\dot{8} = 0.458458458\dots$   
 Let  $x = 0.458458458\dots$   
**Multiply by 1000:**  $1000x = 458.458458458\dots$   
**Subtracting:**  $1000x - x = 458.458458\dots - 0.458458\dots$   
 $999x = 458$   
 $x = \frac{458}{999}$

**E.g. 7** Express  $0.8\dot{5}$  as a fraction in its lowest terms.

**Working:**  $0.8\dot{5} = 0.8555\dots$   
 Let  $x = 0.8555\dots$   
**Multiply by 10 to get recurring part next to decimal point:**  $10x = 8.555\dots$   
**Multiply  $x = 0.8555\dots$  by 100:**  $100x = 85.555\dots$   
**Subtracting:**  $100x - 10x = 85.555\dots - 8.555\dots$   
 $90x = 77$   
 $x = \frac{77}{90}$

**E.g. 8** Express these recurring decimals as fractions in their lowest terms.

(a)  $0.3\dot{2}$                       (b)  $0.94\dot{6} = 0.94666\dots$                       (c)  $0.4\dot{9}\dot{5}$

**Working:** (a)  $0.3\dot{2} = 0.3222\dots$   
 Let  $x = 0.3222\dots$   
**Multiply by 10:**  $10x = 3.222\dots$   
**Multiply  $x = 0.3222\dots$  by 100:**  $100x = 32.222\dots$   
**Subtracting:**  $100x - 10x = 32.222\dots - 3.222\dots$   
 $90x = 29$   
 $x = \frac{29}{90}$

(b)  $0.94\dot{6} = 0.94666\dots$   
 Let  $x = 0.94666\dots$   
**Multiply by 100:**  $100x = 94.666\dots$   
**Multiply  $x = 0.94666\dots$  by 1000:**  $1000x = 946.666\dots$   
**Subtracting:**  $1000x - 100x = 946.666\dots - 94.666\dots$   
 $900x = 852$   
 $x = \frac{852}{900} = \frac{71}{75}$

(c)  $0.4\dot{9}\dot{5} = 0.4959595$   
Let  $x = 0.4959595\dots$

*Multiply by 10:*

*Multiply  $x = 0.4959595\dots$  by 100:*

*Subtracting:*

$$1000x - 10x = 495.959595\dots - 4.959595\dots$$

$$990x = 491$$

$$x = \frac{491}{990}$$

$$10x = 4.959595\dots$$

$$1000x = 495.959595\dots$$

**Video:**

[Recurring decimals to fractions](#)

[Solutions to Starter and E.g.s](#)

**Exercise**

9-1 class textbook:

p10 E1.1 Qu 1-2

A\*-G class textbook:

p10 E1.1 Qu 1-2

9-1 homework book:

p4 E1.1 Qu 1-3

A\*-G homework book:

p3 E1.1 Qu 1-3