Converting Units of Area and Volume

Starter

1. **(Review of last lesson)** A cone has a surface area of 250 m² and its height is 10 m. If the height of a similar cone is 16 m, calculate its surface area.

    Working: Length factor ***from small to big*** $= \dfrac{16}{10} = 1.6$ **small to big so Lf** > 1

    Area factor $=$ Length factor$^2 = 1.6^2 = 2.56$

    Surface area of larger cone $=$ Area factor $\times$ Surface area of A
    $$= 2.56 \times 250$$
    $$= 640 \text{ m}^2$$

2. **(Review of last lesson)** Two spheres are such that their radii (plural of radius) are 7 cm and 12 cm. The larger sphere has a volume of 3600 cm³. Calculate the volume of the smaller sphere. Give your answer to 1 d.p..

    Working: Length factor ***from big to small*** $= \dfrac{7}{12}$ ***big to small so Lf*** < 1

    Volume factor $=$ Length factor$^3 = \left(\dfrac{7}{12}\right)^3 = \dfrac{343}{1728}$

    Volume of big cone $=$ volume factor $\times$ volume of small cone
    $$= \dfrac{343}{1728} \times 3600$$
    $$= 125 \times 7$$
    $$= 714.6 \text{ cm}^3$$

3. Convert 6 m² to cm².

    Working: Many people would ***incorrectly*** suggest that 6 m² is equivalent to 600 cm².
    In fact, the answer is 60000 cm².
    This lesson will explain why.

E.g 1 By using a similar method to above, convert
   (a) 12 m² to cm²   (b) 8 cm² to mm²   (c) 4500 cm² to m²

   Working: (a)   Let the dimensions of the rectangle be 4 m by 3 m.
   Convert to cm: 400 cm by 300 m
   Multiply: 120000 cm²
   So 12 cm² $\equiv 120000$ m²
   Alternatively, your working could look like this:
   12 m² $= 4$ m $\times 3$ m   ***two numbers multiply to get*** 12
   $= 400$ cm $\times 300$ cm   ***convert the units***
   $= 120000$ cm²
   N.B.   I could also have chosen 6 m by 2 m or 12 m by 1 m for the dimensions of the rectangle.

   (b)   Let the dimensions of the rectangle be 8 cm by 1 cm.

        Convert to mm: 80 mm by 10 mm

        Multiply: 800 mm²

        So 8 cm² $\equiv 800$ mm²

        Alternatively, your working could look like this:

$$8 \text{ cm}^2 = 8 \text{ cm} \times 1 \text{ cm} \qquad \textit{two numbers multiply to get } 8$$
$$= 80 \text{ mm} \times 10 \text{ mm} \qquad \textit{convert the units}$$
$$= 800 \text{ mm}^2$$

        N.B.   I could also have chosen 4 cm by 2 cm for the dimensions of the rectangle.

   (c)   Let the dimensions of the rectangle be 45 cm by 100 cm.

        Convert to m: 0.45 m by 1 m

        Multiply: 0.45 m²

        So 4500 cm² $\equiv 0.45$ m²

        Alternatively, your working could look like this:

$$12 \text{ m}^2 = 4 \text{ m} \times 3 \text{ m} \qquad \textit{two numbers multiply to get } 12$$
$$= 400 \text{ cm} \times 300 \text{ cm} \qquad \textit{convert the units}$$
$$= 120000 \text{ cm}^2$$

        N.B.   I could also have chosen 4 cm by 2 cm for the dimensions of the rectangle.

E.g. 2 State what you need to ***multiply*** or ***divide*** by to convert from:

   (a)   cm² to m²   (b)   cm² to mm²   (c)   m² to km²

   Working: (a)   To convert from **cm** to **m** we **divide** by 100.

                         To convert from **cm²** to **m²** we **divide** by 100^2.

             (b)   To convert from **cm** to **mm** we **multiply** by 10.

                         So to convert from **cm²** to **mm²** we **multiply** by 10^2.

             (c)   To convert from **m** to **km** we **divide** by 1000.

                         To convert from **m²** to **km²** we **divide** by 1000^2.

E.g. 3 To convert from **m²** to **cm²** we **multiply** by 100^2 — this is for areas.

   What could we need to multiply by to convert from **m³** to **cm³** (units of volume).

   Working: We would multiply by 100^3 — *cubed* because it is a *volume*.

E.g. 4 State what you need to ***multiply*** or ***divide*** by to convert from:

   (a)   km³ to m³   (b)   mm³ to cm³   (c)   m³ to mm³

   Working: (a)   To convert from **km** to **m** we **multiply** by 1000.

                         So to convert from **km³** to **m³** we **multiply** by 1000^3.

             (b)   To convert from **mm** to **cm** we **divide** by 10.

                         To convert from **mm³** to **cm³** we **divide** by 10^3.

             (c)   To convert from **m** to **cm** we **multiply** by 100.

                         So to convert from **m** to **mm** we **multiply** by 1000.

                         So to convert from **m³** to **mm³** we **multiply** by 1000^3.

E.g. 5 Convert these areas or volumes to the units given:

    (a)   9 m³ to cm³   (b)   72000 mm² to cm²

    (c)   4.3 cm³ to mm³   (d)   3000000 cm² to km²

Working: (a)   To convert from **m** to **cm** we **multiply** by 100.

So to convert from **m³** to **cm³** we **multiply** by 100^3.

$$\therefore 9 \text{ m}^3 \equiv 9 \times 100^3 \text{ cm}^3$$
$$= 9 \times 1000000 \text{ cm}^3$$
$$= 9000000 \text{ cm}^3$$

(b)   To convert from **mm** to **cm** we **divide** by 10.

So to convert from **mm²** to **cm²** we **divide** by 10^2.

$$\therefore 72000 \text{ m}^2 \equiv 72000 \div 10^2 \text{ cm}^2$$
$$= 72000 \div 100 \text{ cm}^2$$
$$= 720 \text{ cm}^2$$

(c)   To convert from **cm** to **mm** we **multiply** by 10.

So to convert from **cm³** to **mm³** we **multiply** by 10^3.

$$\therefore 4.3 \text{ cm}^3 \equiv 4.3 \times 10^3 \text{ mm}^3$$
$$= 4.3 \times 1000 \text{ mm}^3$$
$$= 4300 \text{ mm}^3$$

(d)   To convert from **cm** to **m** we **multiply** by 100.

So to convert from **cm** to **km** we **multiply** by 100000.

So to convert from **cm²** to **km²** we **multiply** by 100000^2.

$$\therefore 3000000 \text{ cm}^2 \equiv 3000000 \div 100000^2 \text{ km}^2$$
$$= 3000000 \div 10000000000 \text{ km}^2$$
$$= 3 \div 10000 \text{ km}^2$$
$$= 0.0003 \text{ km}^2$$

      Video: **Converting between metric units of area**

      Video: **Converting between metric units of volume**

      Solutions to Starter and E.g.s and Exercise

Exercise

1.   Convert these units to the ones stated:

    (a)   9 m² to cm²   (b)   850 mm² to cm²

    (c)   2000000 m³ to km³   (d)   3.65 cm³ to mm³

    (e)   690 cm³ to m³   (f)   5300 km² to m²

    (g)   9100 cm² to m²   (h)   793 km³ to m³

    (i)   4200 mm³ to cm³   (j)   5800 m² to km²

    (k)   6.1 m³ to cm³   (l)   1.8 cm² to mm²

2.   Convert these units to the ones stated:

    (a)   54 km³ to cm³   (b)   850 mm² to m²

Answers

1.   (a)   9 m² ≡ 90000 cm²   (b)   850 mm² ≡ 8.5 cm²

    (c)   2000000 m³ ≡ 0.002 km³   (d)   3.65 cm³ ≡ 3650mm³

    (e)   690 cm³ ≡ 0.00069 m³   (f)   5300 km² ≡ 5300000000 m²

    (g)   9100 cm² ≡ 0.91 m²   (h)   793 km³ ≡ 793000000000 m³

    (i)   4200 mm³ ≡ 4.2 cm³   (j)   5800 m² ≡ 0.0058 km²

    (k)   6.1 m³ ≡ 6100000 cm³   (l)   1.8 cm² ≡ 180 mm²

2.   Convert these units to the ones stated:

    (a)   54 km² ≡ 540000000000 cm²   (b)   9 m³ ≡ 9000000000 mm³