Equation of a Line Given Two Points

Starter

1. **(Review of last lesson)** State the gradient and $y-$intercept of the following:
(a) $y = 6x + 5$ (b) $y = 9 - x$ (c) $y + 7 = x$

Working: (a) Gradient $= 6$ $y-$intercept $= 5$

(a) Gradient $= -1$ $y-$intercept $= 9$
N.B. Remember the gradient is the coefficient of x and the $y-$intercept is the constant term.

(a) The equation needs to be rearranged into the form $y = mx + c$
Gradient $= 6$ $y-$intercept $= 5$

2. **(Review of last lesson)**
State the equation of the line which has gradient 2 and $y-$intercept -3.

Working: $y = 2x - 3$

E.g. 1 Find the gradient of the line passing through the following pairs of points:
(a) $(2, 4)$ and $(5, 19)$ (b) $(-3, 2)$ and $(5, 4)$

Working: (a) ***Label the points:*** $(2, 4)$ $(5, 19)$
(x_1, y_1) (x_2, y_2)

Sub. into formula: Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{19 - 4}{5 - 2} = \dfrac{15}{3} = 5$

(b) ***Label the points:*** $(-3, 2)$ $(5, 4)$
(x_1, y_1) (x_2, y_2)

Sub. into formula: Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 2}{5 - -2} = \dfrac{2}{8} = \dfrac{1}{4}$

E.g. 2* The gradient of a line passing through the point $(-3, 9)$ and $(p, 4)$ is 6. Find the value of p.

Working: ***Label the points:*** $(-3, 9)$ $(p, 4)$
(x_1, y_1) (x_2, y_2)

Sub. into formula Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$: $6 = \dfrac{4 - 9}{p - -3}$

$$6 = \frac{-5}{p + 3}$$

Multiply by $(p + 3)$***:*** $6(p + 3) = -5$
Expand the brackets: $6p + 18 = -5$
Subtract 18 ***from both sides:*** $6p = -23$

$$p = -\frac{23}{6}$$

E.g. 3 Find the equation of the line that has the gradient and passes through the point given:

(a) gradient $= 3$, passes through $(4, 5)$

(b) gradient $= -2$, passes through $(7, -1)$

Working:

(a) **Substitute the gradient into** $y = mx + c$**:** $y = 3x + c$

Substitute $(4, 5)$ ***into*** $y = 3x + c$***:***

$$5 = 3 \times 4 + c$$
$$5 = 12 + c$$
$$c = -7$$

The equation of the line is $y = 3x - 7$

(b) **Substitute the gradient into** $y = mx + c$**:** $y = -2x + c$

Substitute $(7, -1)$ ***into*** $y = -2x + c$***:***

$$-1 = -2 \times 7 + c$$
$$-1 = -14 + c$$
$$c = 13$$

The equation of the line is $y = -2x + 13$

E.g. 4 Find the equation of the line passing through

(a) $(2, 1)$ and $(4, 11)$ (b) $(2, 3)$ and $(6, -5)$

Working:

(a) Label the points: $(2, 1)$ $(4, 11)$

(x_1, y_1) (x_2, y_2)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 1}{4 - 2} = \frac{10}{2} = 5$$

Substitute the gradient into $y = mx + c$**:** $y = 5x + c$

Substitute $(2, 1)$ ***into*** $y = 5x + c$***:***

$$1 = 5 \times 2 + c$$
$$1 = 10 + c$$
$$c = -9$$

The equation of the line is $y = 5x - 9$

(b) Label the points: $(2, 3)$ $(6, -5)$

(x_1, y_1) (x_2, y_2)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 3}{6 - 2} = \frac{-8}{4} = -2$$

Substitute the gradient into $y = mx + c$**:** $y = -2x + c$

Substitute $(2, 3)$ ***into*** $y = -2x + c$***:***

$$3 = (-2) \times 2 + c$$
$$3 = -4 + c$$
$$c = 7$$

The equation of the line is $y = -2x + 7$

Video: Finding the equation of a straight line passing through two points

Solutions to Starter and E.g.s

Exercise

p51 Ex 14.5 Qu 1-7

Textbook answers (only available during a lockdown)