Independent Events and Tree Diagrams

Starter

1. **(Review of last lesson)**
Stella and Paul each have three cards numbered 1, 2 and 3. They each take one of the other's cards and add the total. What is the probability the total will be an odd number?
Hint: list the possible outcomes in a probability space diagram.

Working: Odd + Odd = Even
Odd + Even = Odd
Even + Even = Even
From this we can draw the table:

	1	2	3
1	e	o	e
2	o	e	o
3	e	o	e

$$P(\text{odd}) = \frac{4}{9}$$

2. **(Review of previous material)**
Without using a calculator, find: (a) $\frac{6}{11} \times \frac{5}{11}$ (b) $\frac{4}{9} \times \frac{3}{5}$

Working: (a) $\frac{6}{11} \times \frac{5}{11} = \frac{6 \times 5}{11 \times 11} = \frac{30}{121}$

(b) $\frac{4}{9} \times \frac{3}{5} = \frac{4}{3} \times \frac{1}{5} = \frac{4}{15}$
N.B. Cancel top and bottom before multiplying.

3. **(Review of previous material)**
Without using a calculator, find: (a) 0.6×0.4 (b) 0.7×0.1

Working: (a) $0.6 \times 0.4 = 0.24$

(b) $0.7 \times 0.1 = 0.07$

E.g. 1 Decide whether the following events are independent or not.

(a) Flipping two coins.
(b) Penalties in a penalty shoot-out.
(c) Rolling two coins.
(d) The weather on two consecutive days.
(e) The weather on days separated by 6 months.

Working:

(a) Independent.

(b) Not independent since pressure may increase during the shoot-out making it less likely to score. Also different players are taking the penalties.

(c) Independent, although you could argue that they shouldn't touch each other as they roll.

(d) Not independent since the weather on the first day affects the weather on the second day.

(e) Independent.

E.g. 2 The probability that Irina is late for school is 0.3; the probability that Helena is late is 0.2. What is the probability that both girls are late for school?

Working: $P(\text{late, late}) = P(\text{late}) \times P(\text{late}) = 0.3 \times 0.2 = 0.06$

E.g. 3 A coin is flipped 3 times. What is the probability of getting 3 heads?

Working:

$$\begin{aligned} P(\text{head, head, head}) &= P(\text{head}) \times P(\text{head}) \times P(\text{head}) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \\ &= \frac{1}{8} \end{aligned}$$

E.g. 4 A dice is rolled twice. Find the probability of getting a two on just one of these rolls.

Working: $P(\text{a 2}) = \frac{1}{6}$ so $P(\text{not a 2}) = \frac{5}{6}$

$$\begin{aligned} P(\text{a 2 on just one roll}) &= P(\text{a 2, not a 2}) + P(\text{not a 2, a 2}) \\ &= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \\ &= \frac{5}{36} + \frac{5}{36} \\ &= \frac{10}{36} = \frac{5}{18} \end{aligned}$$

E.g. 5 While driving to work, Sarah passes through two sets of traffic lights. The probability the first set is green is 0.4 and the probability the second is green is 0.3.

(a) Draw a tree diagram.

(b) Find the probability that:

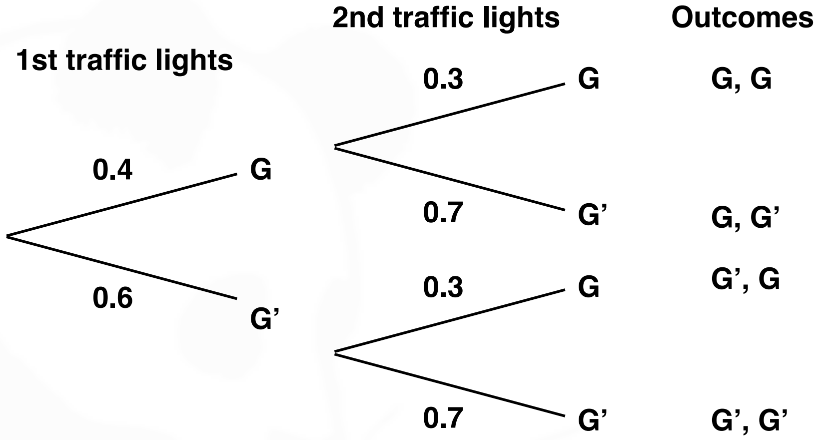
(i) neither set of lights is green

(ii) only one set of traffic lights is green

(iii) at least one set of traffic lights is green

Hint: In your tree diagram use G for "green" and G' for "not green".

Working: (a)



(b) (i) $P(\text{neither set of lights is green}) = P(G', G')$

$= P(G') \times P(G')$

$= 0.6 \times 0.7$

$= 0.42$

(ii) $P(\text{1 set is green}) = P(G, G') + P(G', G)$

$= P(G) \times P(G') + P(G') \times P(G)$

$= 0.4 \times 0.7 + 0.6 \times 0.3$

$= 0.28 + 0.18$

$= 0.46$

(iii) $P(\text{at least 1 set is green}) = 1 - P(\text{no sets are green})$

$= 1 - P(G', G')$

$= 1 - P(G') \times P(G')$

$= 1 - 0.6 \times 0.7$

$= 1 - 0.42$

$= 0.58$

N.B. For (iii) you could also do:

$P(\text{at least 1 set is green}) = P(\text{1 set is green}) + P(\text{2 sets are green})$

Video: Independent events

Video: Tree diagrams

Solutions to Starter and E.g.s

Exercise

p181 Ex 10.4 Qu 2, 3, 5, 7-15

Textbook answers (only available during a lockdown)