Length, Area and Volume Scale Factors

Starter

1. A square of length 5 units is enlarged by a length factor of 3.
   (a) Find the area:
      (i) before and
      (ii) after the enlargement.
   (b) State the area factor from the small square to the big square.
   (c) What is the connection between the length factor of 3 and the area factor you calculated?

Working:

(a) (i) $5^2 = 25$

(ii) New side length $= 5 \times 3$

So new area $= 15^2 = 225$

(b) Small to big is from 25 to 225

So the area factor from small to big $= \dfrac{225}{15} = 9$

(c) $3^2 = 9$

$$\text{Area factor} = \text{Length factor}^2$$

E.g. 1 Two triangles, P and Q, are similar. Triangle P has base length 2 cm and area 7 cm^2, while Q has base length 8 cm. Find the area of triangle Q.

Working:

Length factor ***from P to Q*** $= \dfrac{8}{2} = 4$ ***small to big so Lf*** > 1

Area factor $=$ Length factor$^2 = 4^2 = 16$

Area of triangle Q $=$ Area factor $\times$ Area of P

$= 16 \times 7$

$= 112$ cm^2

E.g. 2 Cylinders A and B are similar shapes and their radii are 5 cm and 4 cm respectively. Given that cylinder A has surface area 100 cm^2, find the surface area of B.

Working:

Length factor ***from A to B*** $= \dfrac{4}{5}$ **big to small so Lf** < 1

Area factor $=$ Length factor$^2 = \left(\dfrac{4}{5}\right)^2 = \dfrac{16}{25}$

Surface area of cylinder B $=$ Area factor $\times$ Area of A

$= \dfrac{16}{25} \times 100$

$= 64$ cm^2

Volume factors

E.g. 3 We know that if the length factor is k, the ***area*** factor is k^2.
What would be the ***volume*** factor?

Working: The volume factor would be k^3.

$$\text{Volume factor} = \text{Length factor}^3$$

E.g. 4 The heights of two similar cuboids are 3 m and 6 m respectively. The smaller cuboid has a volume of 45 m³. Calculate the volume of the larger cuboid.

Working:

Length factor ***from small to big*** $= \frac{6}{3} = 2$ ***small to big so Lf > 1***

Volume factor $=$ Length factor$^3 = 2^3 = 8$

Volume of big cuboid $=$ volume factor $\times$ volume of small cuboid

$= 45 \times 8$

$= 360$ m³

E.g. 5 The diameters of the bases of two similar cones are 8 cm and 20 cm. Given that the smaller cone has volume 56 cm³, find the volume of the other cone.

Working:

Length factor ***from small to big*** $= \frac{20}{8} = \frac{5}{2}$ ***small to big so Lf > 1***

Volume factor $=$ Length factor$^3 = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

Volume of big cone $=$ volume factor $\times$ volume of small cone

$= \frac{125}{8} \times 56$

$= 125 \times 7$

$= 875$ cm³

Video: Similar shapes - areas
Video: Similar solids - volumes

Solutions to Starter and E.g.s

Exercise
p145 Ex 19.3 Qu 4-10