Angles in regular polygon

Starter

1. **(Review of last lesson)** A dodecagon has 5 interior angles each measuring $140°$ and 4 interior angles each measuring $160°$. The remaining angles are all equal — find the size of one of the remaining angles.

N.B. A regular polygon is one whose sides are all equal in length and whose angles are all equal.

2. The sum of the exterior angles of any polygon is $360°$.
Write down the formula for the size of one exterior angle of an $n-$sided regular polygon.

Working:

An $n-$sided regular polygon has n equal angles.

Size of one exterior angle of an $n-$sided regular polygon $= \dfrac{360°}{n}$

Notes

Here are the complete facts and formulae for polygons and regular polygons:

Exterior angle $+$ Interior angle $= 180°$

Sum of ***interior*** angles of an $n-$sided polygon $= 180°(n-2)$

Size of each ***interior*** angle of an $n-$sided ***regular*** polygon $= \dfrac{180°(n-2)}{n}$

Sum of ***exterior*** angles of any polygon is $360°$

Size of each ***exterior*** angle of an $n-$sided ***regular*** polygon $= \dfrac{360°}{n}$

E.g. 1 Find the size of each exterior angle for:
(a) an $11-$sided regular polygon (b) a $17-$sided regular polygon.

Working: (a) Size of each interior angle of an $11-$sided regular polygon $= \dfrac{360°}{n}$

$$= \frac{360°}{11}$$
$$\approx 32.7°$$

E.g. 2 How many sides does the regular polygon have given that each ***exterior*** angle is:
(a) $24°$ (b) $8°$

Working: (a) ***Each exterior angle*** $= \dfrac{360°}{n}$***:*** $24° = \dfrac{360°}{n}$

$$n = \frac{360°}{24°} = 15 \text{ sides}$$

E.g. 3 How many sides does the regular polygon have given that each ***interior*** angle is:
(a) 140° (b) 170°

Working: (a) Exterior angle $= 180^\circ - 140^\circ = 40^\circ$

Each exterior angle $= \frac{360^\circ}{n}$**:** $40^\circ = \frac{360^\circ}{n}$

$n = \frac{360^\circ}{40^\circ} = 9$ sides

E.g. 4 Mary claims that she has found a regular polygon whose exterior angles are all 27°. Is she correct? Show your working.

E.g. 5 Abdul claims that he has found a regular polygon whose interior angles are four times the size of the exterior angle. If the polygon exists, how many sides does it have?

Video: Angles in polygons

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p68 M3.4 Qu 1-8 even, 9-16
A*-G class textbook:	p61 M3.4 Qu 1-15
9-1 homework book:	p22 M3.4 Qu 1-12
A*-G homework book:	p16 M3.4 Qu 1-9

Summary

Exterior angle $+$ Interior angle $= 180^\circ$

Sum of ***interior*** angles of an $n-$sided polygon $= 180^\circ(n-2)$

Size of each ***interior*** angle of an $n-$sided ***regular*** polygon $= \frac{180^\circ(n-2)}{n}$

Sum of ***exterior*** angles of any polygon is 360°

Size of each ***exterior*** angle of an $n-$sided ***regular*** polygon $= \frac{360^\circ}{n}$

n-sided polygon	Sum of angles	Size of one angle in a regular polygon
Interior angles	$180(n-2)$	$\frac{180(n-2)}{n}$
Exterior angles	360°	$\frac{360}{n}$