Calculations with error intervals

Starter

1. **(Review of last lesson)** Write down the error interval of 7200 when it has been:
   (a) rounded to 2 s.f. (b) truncated to 2 s.f.

2. Two numbers, x and y, rounded to the nearest whole number become 7 and 4 respectively.
   (a) Write down the error interval for x and the error interval for y.
   (b) State the smallest and largest possible values of $x + y$.
   Write down your calculation.
   (c) State the smallest and largest possible values of $x - y$.
   Write down your calculation.
   (d) State the smallest and largest possible values of $x \times y$.
   Write down your calculation.
   (e) State the smallest and largest possible values of $x \div y$.
   Write down your calculation.

Notes

Before finding the upper and lower bounds of calculation, the upper and lower bounds of the individual numbers must be found.

E.g. 1 Using your answers to the starter, copy and complete the table below:

x and y	Lower bound	Upper bound
Addition	$\text{lb}(x) + \text{lb}(y)$	
Subtraction		
Multiplication		
Division		

N.B. Notice the similarity between ***addition/multiplication*** and ***subtraction/division***.

E.g. 2 If $a = 3.1$ and $b = 8.4$, both correct to 1 d.p., find:
(a) the upper bound of $a + b$ (b) the lower bound of $b - a$
(c) the lower bound of ab (d) the upper bound of $\frac{b}{a}$

Working: (a) The error interval of $a = 3.1$ is $3.05 \leq a < 3.15$
The error interval of $b = 8.4$ is $8.35 \leq a < 8.45$
Upper bound of $a + b = \text{ub}(a) + \text{ub}(b) = 3.15 + 8.45 = 11.6$

E.g. 3 The radius of a circle, r, is given as 6.83 to 2 d.p. Calculate the error interval of:
(a) the circumference (b) the area.
Give your answers to 4 s.f..

Working: (a) The error interval of 6.83 is $6.825 \leq r < 6.835$
Lower bound $= 2 \times \pi \times 6.825 = 42.88$
Upper bound $= 2 \times \pi \times 6.835 = 42.95$
Error interval: $42.88 \leq$ Circumference < 42.95

E.g. 4 Abdul is travelling from Ripon to London. When he set off, his GPS said it was a journey of 230 miles, to the nearest 5 miles. After a few hours the speedometer in his car indicates he has covered 132 miles, to the nearest mile. What is the least distance he has left to travel?

Safety questions

E.g. 5 A notice in a lift in Brazil says that it can carry 1600 kg safely. This figure is correct to the nearest 100 kg. The weight of the average person in Brazil is 68 kg to the nearest kilogram. What is the maximum number of people that the lift can ***safely*** hold? Give a reason why your value may be too high.

Working:

Error interval of lift: $1550 \leq x < 1625$

Error interval of person's weight: $67.5 \leq y < 68.5$

Think "worst case scenario".

Maximum safe number of people $\equiv$ lower bound

Maximum safe number of people $= \dfrac{\text{lb}(1600)}{\text{ub}(68)} = \dfrac{1550}{68.5} \approx 22.6$

The maximum number of people that the lift can ***safely*** hold is 22.

The value may be too high if 22 people enter the lift who are all above average weight.

When ***safety questions*** ask for the "maximum value that can be lifted safely…" they are actually asking for the ***lower bound***, despite the use of the word "maximum".

Hint: Think "***worst case scenario***".

E.g. 6 A crane has a cable with a breaking strain of 5300 kg measured to 2 significant figures. It is used to lift crates which weigh 100 kg measured to the nearest 10 kg. What is the greatest number of crates that can be lifted so that the cable will definitely not break?

Video 1: Calculations with error intervals
Video 2: Calculations with error intervals

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p139 E5.1 Qu 1-13
A*-G class textbook:	p130 E5.1 Qu 1-10
9-1 homework book:	p49 E5.1 Qu 1-9
A*-G homework book:	p36 E5.1 Qu 1-6

Summary

x and y	Lower bound	Upper bound
Addition	$\text{lb}(x) + \text{lb}(y)$	$\text{ub}(x) + \text{ub}(y)$
Subtraction	$\text{lb}(x) - \text{ub}(y)$	$\text{ub}(x) - \text{lb}(y)$
Multiplication	$\text{lb}(x) \times \text{lb}(y)$	$\text{ub}(x) \times \text{ub}(y)$
Division	$\text{lb}(x) \div \text{ub}(y)$	$\text{ub}(x) \div \text{lb}(y)$