

Equation of a straight line

Starter

- (Review of last lesson)**
Consider the line segment joining the points $(-3, 4)$ and $(2m, n)$.
 - Find an expression for the gradient of the line segment in terms of m and n
 - Calculate the value of m or n given that:
 - the line is parallel to the y -axis,
 - the line is parallel to the x -axis.
- (Review of previous material)** State the gradient and y -intercept of these lines:
 - $y = 4x + 7$
 - $y = 9 - 5x$
 - $x + y + 3 = 0$

Notes

Last academic year, you learnt that $y = mx + c$ is the equation of a straight line where
 m is the gradient
 and c is the y -intercept.

N.B. If the equation is not in the form $y = mx + c$, rearrange the equation so that $y = \dots$ before stating the gradient and the y -intercept.

E.g. 1 State the gradient and y -intercept of these lines:

- $6x + 3y = 2$
- $5y - 6x - 15 = 0$
- $y = 7$
- $x = -4$

Working:

| | | |
|-----------------|--|------------------------|
| (a) | Subtract $6x$ from both sides: | $6x + 3y = 2$ |
| | | $3y = 2 - 6x$ |
| | Divide both side by 3: | $y = \frac{2}{3} - 2x$ |
| Gradient = -2 | y -intercept = $\frac{2}{3}$ | |

Finding the equation of a straight line given the gradient and one point

If we know the gradient of a line and a point that it passes through, we can find its equation.

Success Criteria

- Substitute the value for the gradient into the equation $y = mx + c$.
- Substitute the x -value and the y -value of the point into the equation from step 1 and solve the equation to find the value of c .
- Replace c by the value found in step 2 in $y = mx + c$.

E.g. 2 Find the equation of the line that has the gradient and passes through the point given:

- gradient = -4 , passes through $(-2, 7)$
- gradient = $\frac{1}{3}$, passes through $(6, -5)$
- gradient = m , passes through (x_1, y_1)

Working:

| | | |
|-----|---|--------------------------|
| (a) | Substitute the gradient into $y = mx + c$: | $y = -4x + c$ |
| | Substitute $(-2, 7)$ into $y = -4x + c$: | $7 = -4 \times (-2) + c$ |
| | | $7 = 8 + c$ |
| | | $c = -1$ |

The equation of the line is $y = -4x - 1$

Instead of using the method described above the number for the gradient and the point could be substituted into the formula:

$$y - y_1 = m(x - x_1)$$

E.g. 3 Using the formula $y - y_1 = m(x - x_1)$, find the equation of the straight line which has:

- (a) gradient = 2, passes through (3, -4)
- (b) gradient = -5, passes through (-1, 8)

Working: (a) $y - y_1 = m(x - x_1)$: $y - (-4) = 2(x - 3)$
 $y + 4 = 2x + 6$
 $y = 2x - 2$

(b) $y - y_1 = m(x - x_1)$: $y - 8 = -5(x - (-1))$
 $y - 8 = -5(x + 1)$
 $y - 8 = -5x - 5$
 $y = -5x + 3$

Finding the equation of the line passing through two points

Success Criteria

1. Find the gradient between the 2 points using Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$
2. Substitute the value for the gradient into the equation $y = mx + c$.
3. Choose either of the given points. Substitute the x -value and the y -value of the point into the equation from step 1 and solve the equation to find the value of c .
4. Replace c by the value found in step 2 in $y = mx + c$.

E.g. 4 Find the equation of the line passing through

- (a) (5, 1) and (2, -5)
- (b) (2, -1) and (4, -9)

Working: (a) Label the points: (5, 1) (2, -5)
 (x_1, y_1) (x_2, y_2)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 1}{2 - 5} = \frac{-6}{-3} = 2$$

Substitute the gradient into $y = mx + c$: $y = 2x + c$

Substitute (5, 1) into $y = 2x + c$: $1 = 2 \times 5 + c$

$$1 = 10 + c$$

$$c = -9$$

The equation of the line is $y = 2x - 9$

Video: [y = mx + c](#)

Video: [Finding the equation of a straight line](#)

Video: [Finding the equation of a straight through 2 points](#)

[Solutions to Starter and E.g.s](#)

Exercise

- 9-1 class textbook: p196 M6.14 Qu 1-20 odd, 25, 27-36, 39
- A*-G class textbook: p180 M6.12 Qu 1-20 odd, 25, 27-36
- 9-1 homework book: p70 M6.14 Qu 1-10
- A*-G homework book: p53 M6.12 Qu 1-8

Summary

$y = mx + c$ is the equation of a straight line where m is the gradient and c is the y -intercept.

Finding the equation of a straight line given the gradient and one point:

1. Substitute the value for the gradient into the equation $y = mx + c$.
2. Substitute the x -value and the y -value of the point into the equation from step 1 and solve the equation to find the value of c .
3. Replace c by the value found in step 2 in $y = mx + c$.

Finding the equation of the line passing through two points:

1. Find the gradient between the two points using Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$
2. Substitute the value for the gradient into the equation $y = mx + c$.
3. Choose either of the given points. Substitute the x -value and the y -value of the point into the equation from step 1 and solve the equation to find the value of c .
4. Replace c by the value found in step 2 in $y = mx + c$.