# Equations of Motion (constant acceleration)

**Starter**

1. Given that $v^2 = u^2 + 2as$, find the possible values of $v$ when $u = -3$, $a = 5$ and $s = 4$.

2. Rearrange the formula $s = ut + \frac{1}{2}at^2$ to make $a$ the subject.

**Notes**

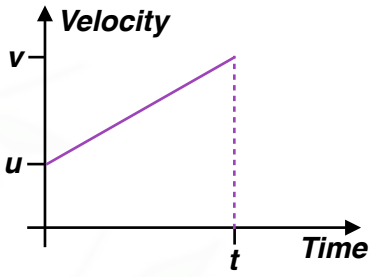
The equations of motion, where ***acceleration is constant***, use the letters $u, v, a, s$ and $t$.

$u$ = initial velocity
$v$ = final velocity
$a$ = acceleration
$s$ = displacement
$t$ = time

***E.g. 1*** Consider the velocity-time graph.

(a) Using the fact that the gradient of a line is the acceleration, find an equation involving $u$, $v$, $a$ and $t$.
Rearrange your equation to make $v$ the subject.

(b) Using the fact that the area under the line is the distance travelled, find an equation involving $s$, $u$, $v$ and $t$.




***Working:*** (a) Gradient $\equiv a = \frac{v-u}{t}$

***Multiplying both side by $t$:*** $at = v - u$

***Make $v$ the subject:*** $v = u + at$ No $s$

Further equations can be derived by replacing a letter in $s = \frac{1}{2}(u+v)t$ by an expression from $v = u + at$.

***Replacing $t$***

We can rearrange $v = u + at$ to make $t$ the subject: $v = u + at$

***Subtract $u$ from both sides:*** $v - u = at$

***Divide both sides by $a$:*** $t = \frac{v-u}{a}$

Replace $t$ by $\frac{v-u}{a}$ in $s = \frac{1}{2}(u+v)t$: $s = \frac{1}{2}\left(u+v\right)\left(\frac{v-u}{a}\right)$

***Expand the brackets:*** $s = \frac{1}{2}\left(\frac{v^2-u^2}{a}\right)$

***Expand the bracket:*** $s = \frac{v^2-u^2}{2a}$

***Multiply both sides by $2a$:*** $2as = v^2 - u^2$

***Add $u^2$ to both sides:*** $v^2 = u^2 + 2as$ No $t$

***E.g. 2*** By replacing $v$ by $u + at$ in $s = \frac{1}{2}(u + v)t$, find a formula for $s$ in terms of $a$, $t$ and $u$.

The equations of motion required for GCSE maths are:

| | |
|---|---|
| $v = u + at$ | No $s$ in the equation |
| $s = ut + \frac{1}{2}at^2$ | No $v$ in the equation |
| $v^2 = u^2 + 2as$ | No $t$ in the equation |

***N.B.*** The equations of motion are often called the ***SUVAT*** equations.

***Success Criteria***

1. Write down which letters you are given and which one you need to find.
2. Write down which letter you do not have/need.
3. Choose the correct equation.
4. Substitute the values you have into the equation and, if necessary, solve the equation to find the required value.

***E.g. 3*** Choose the correct formula in order to calculate the missing value.
***Do not calculate the missing value.***

(a) $a = 3, u = 2, v = 9$ find $t$

(b) $s = 50, v = 11, a = 2$ find $u$

(c) $u = 1, a = 2.5, t = 6$ find $s$

(d) $u = 8, t = 6, s = 100$ find $a$

(e) $u = 3, a = 2, s = 38$ find $v$

(f) $v = 10, u = 6, a = 1.5,$ find $t$

***Working:*** (a) No $s$ $\Rightarrow$ $v = u + at$

***E.g. 4*** Find $v$ given that $a = 5, u = 4$ and $s = 2$

***Working:*** ***No $t$ so choose the equation without $t$:***

$$v^2 = u^2 + 2as$$

***Substitute the values:***

$$v^2 = 4^2 + 2 \times 5 \times 2$$
$$v^2 = 36$$
$$v = 6$$

***E.g. 5*** A car accelerates from $20$ m/s to $35$ m/s in $3$ seconds. Find the acceleration.

***E.g. 6*** A car has initial velocity $30$ km/h and accelerates at $4$ m/s² for $12$ seconds. Find the distance travelled.

**Video:** SUVAT equations and examples

Video: SUVAT examples

Solutions to Starter and E.g.s

**Exercise**

| | |
|---|---|
| 9-1 class textbook: | p211 E6.12 Qu 1-8, 10 |
| A*-G class textbook: | No exercise |
| 9-1 homework book: | p76 E6.12 Qu 1-8 |
| A*-G homework book: | No exercise |

**Summary**

The equations of motion required for GCSE maths are:

| | |
|---|---|
| $v = u + at$ | No $s$ in the equation |
| $s = ut + \frac{1}{2}at^2$ | No $v$ in the equation |
| $v^2 = u^2 + 2as$ | No $t$ in the equation |

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