Error intervals

Starter

1. **(Review of last lesson)** The petrol consumption of a car is 14.8 miles per litre and petrol costs £1.29 per litre. Jasper ***estimates*** that the petrol costs of a round trip of about 4531 miles will be £400. Is this a reasonable estimate? Show your working.

2. Write down (i) the lowest number and (ii) the highest number that rounds to:
(a) 360 (nearest 10) (b) 36 (nearest integer) (c) 36.4 (1 d.p.)

Notes

Error intervals for rounded numbers

When a measurement is given as 145 cm to the nearest centimetre, it means that the length could be anywhere between 144.5 cm and $145.4999\ldots$cm. This range of values is called the ***error interval*** or ***limit of accuracy***.

144.5 cm is called the ***lower bound***.

The value of $145.4999\ldots$ is inconvenient for the upper bound so we use 145.5 instead.

When expressed as an inequality, it is 144.5 cm $\leq$ length < 145.5 cm.

Diagram for an error interval

Inequalities can be shown on a number line but we must distinguish between inequalities that include the number, $\leq$ and $\geq$, and those that don't include the number, $<$ and $>$.

- For $\leq$ and $\geq$: use a ball ●
- For $<$ and $>$: use a circle ○

So the error interval 144.5 cm $\leq$ length < 145.5 cm would look like:

There are two methods to find the error intervals of a number.

Success criteria 1 — next number up/down method

1. Write down the next number down to the given degree of accuracy.
2. The lower bound is half way between the number and the next number down.
3. Write down the next number up to the given degree of accuracy.
4. The upper bound is half way between the number and the next number up.

E.g. Find the error interval of 25.7 rounded to 1 d.p..

Working: The next number down to 1 d.p. is 25.6 so the lower bound is 25.65.
The next number up to 1 d.p. is 25.8 so the upper bound is 25.75.
The error interval is $25.65 \leq x < 25.75$

Success criteria 2 — half a unit method
Lower bound = measurement — half a unit
Upper bound = measurement + half a unit

E.g. Find the error interval of 85 rounded to the nearest 5.

Working: The unit is 5 so half a unit is 2.5
Lower bound $= 85 - 2.5 = 82.5$
Upper bound $= 85 + 2.5 = 87.5$
The error interval is $82.5 \leq x < 87.5$

E.g. 1 Write down the error intervals of:
(a) 18 (rounded to the nearest integer)
(b) 8300 (rounded to the nearest 100)
(c) 9.36 (rounded to 2 d.p.)
(d) 70 (rounded to 1 s.f.)

Working: (a) 18 (rounded to the nearest integer)
Next integer down is 17, next integer up is 19.
The error interval is $17.5 \leq x < 18.5$

Error intervals for truncated numbers
Truncation is an alternative way to rounding that approximates numbers. Numbers are truncated to a certain number of decimal places or significant figures.

When a number is truncated, the extra digits

- are ***removed if*** they were ***after the decimal point***
- ***become zeros if*** they were ***before the decimal point***.

This is similar to rounding to significant figures.

E.g. 5.789 truncated to 1 d.p. is 5.7
767093 truncated to 2 s.f. is 760000

N.B. When truncating, digits are not rounded up.

E.g. 2 Truncate these numbers to the required degree of accuracy:
(a) 7.3198 to 2 d.p. (b) 893526 to 1 s.f.

Working: (a) 7.3198 truncated to 2 d.p. is 7.31

E.g. 3 State the smallest and largest numbers that truncate to 6.34 to 2 d.p.. Hence write down the error interval for numbers truncated to 6.34 to 2 d.p..

Truncating: the lower bound is the number itself
the upper bound is the next number up to that level of accuracy.

Comparing truncating and rounding
The error interval for a 6.34 truncated to 2 d.p. is $6.34 \leq x < 6.35$
The error interval for a 6.34 rounded to 2 d.p. is $6.335 \leq x < 6.345$.

When ***truncating*** the ***original number*** is at the ***start of the error interval***, whereas with ***rounding*** it is in the ***middle of the error interval***.

E.g. 4 State the error intervals for:

(a) 36 (truncated to 2 s.f.)

(b) 5.4 (truncated to 1 d.p.)

(c) 34.10 (truncated to 2 d.p.)

(d) 40 (truncated to 1 s.f.)

Working: (a) Lower bound is the number itself i.e. 36
Upper bound is the next number up 37
The error interval is $36 \leq x < 37$

Video: **Error intervals - rounding and truncating**

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p136 M5.6 Qu 1-15
A*-G class textbook:	p128 M5.6 Qu 1-12
9-1 homework book:	p48 M5.6 Qu 1-14
A*-G homework book:	p35 M5.6 Qu 1-10

Summary

Error intervals for rounded numbers

Either: next number up, next number down — the upper and lower bounds are the mid-points.

…or… Lower bound = measurement — half a unit
Upper bound = measurement + half a unit

Error intervals for truncated numbers

The lower bound is the number itself
The upper bound is the next number up to that level of accuracy.