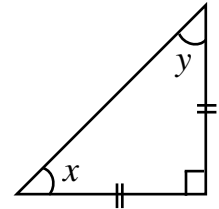


Exact Trigonometric Values

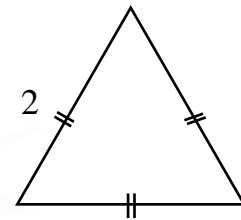
Starter

1. (Review of last lesson) Rationalise and simplify $\frac{\sqrt{5}}{4\sqrt{10}}$.

2. Consider the right-angled isosceles triangle with angles.
- State the size of angles x and y .
 - Given that the length of the opposite and adjacent sides are 1 unit, calculate the length of the hypotenuse.
 - Hence write down the values of $\tan 45^\circ$, $\sin 45^\circ$ and $\cos 45^\circ$. Rationalise the denominator where necessary.



3. Draw an equilateral triangle of side 2 units.
- Calculate the height of the triangle.
 - Hence write down the values of:
 - $\tan 60^\circ$, $\sin 60^\circ$ and $\cos 60^\circ$
 - $\tan 30^\circ$, $\sin 30^\circ$ and $\cos 30^\circ$
 Rationalise the denominator where necessary.



Notes

E.g. 1 Collate the values from the starter in the table below:

	30°	45°	60°
sin			
cos			
tan			

Write down what you notice about the values.

Working:

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Notice that the cosine values are the sine values in reverse.

To get the tangent values do $\frac{\sin}{\cos}$:

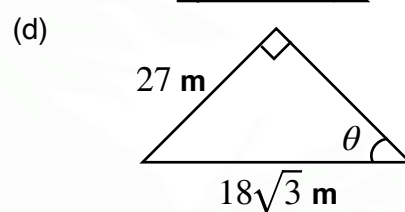
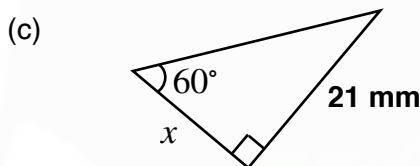
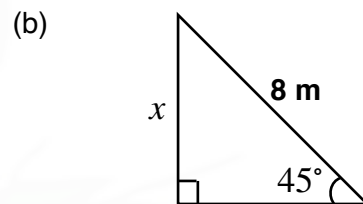
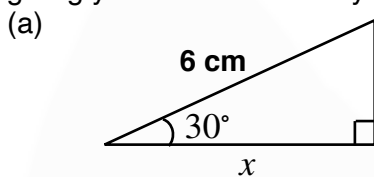
E.g. $\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

N.B. The values in the tables are not given in the exam so you need to memorise them.

When calculating with angles of 30° , 45° and 60° it is more accurate to use the exact value and leave the final answer in surd form. If the answer is then used in a further calculation, it will not bring in a rounding error.

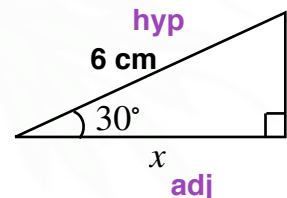
To solve problems use the same method learnt previously but rather than using a calculator, substitute the values from the table.

E.g. 2 Without using a calculator, find the marked length or angle in these right-angled triangles, giving your answers exactly and rationalising surds where needed:



Working:

(a) adj and hyp \Rightarrow cos
 $\cos \theta = \frac{\text{adj}}{\text{hyp}}$: $\cos 30 = \frac{x}{6}$
 $6 \cos 30 = x$
 But $\cos 30 = \frac{\sqrt{3}}{2}$: $x = 6 \times \frac{\sqrt{3}}{2}$
 $x = 3\sqrt{3} \text{ cm}$



Video: [Exact trigonometric values](#)

[Solutions to Starter and E.g.s](#)

Exercise

- 9-1 class textbook: Non-calculator p326 M10.10 Qu 1-12 Draw all diagrams
- A*-G class textbook: No exercise
- 9-1 homework book: Non-calculator p113 M10.10 Qu 1-8 Draw all diagrams
- A*-G homework book: No exercise

Summary

Exact trigonometric values allow the answer to be expressed exactly, as a surd.

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$