Expressions, equations and identities

Starter

1.   **(Review of last lesson)**
      Expand and simplify: (a) $(3x - 1)(4x + 5)$ (b) $3(5x + 2)(x - 4)$

2.   **(Review of last lesson)**
      Expand and simplify   (a) $(x - 4)^2$ (b) $(4x + 3)^2$

Notes

Equals symbol: $=$
Equivalent symbol: $\equiv$

An *expression* does not contain an '$=$' or an '$\equiv$' symbol.
      E.g. $3x - 2$ and $(x + 5)(x - 8)$ are both expressions

An *equation* contains an '$=$' symbol and is *only true for specific values* of the unknown letter.
      E.g. $3x - 2 = 16$ is an equation and is only true for $x = 6$
              $x^2 - 16 = 0$ is an equation and is only true for $x = 4$ and $x = -4$

N.B.   **LHS** is short for *left-hand side* and **RHS** is short for *right-hand side*.

An *identity* is a relationship which is *true for all values* of the unknown (true for all values i.e. LHS is the same as the RHS but just written in a different way). Strictly speaking, the equivalent symbol '$\equiv$' should be used in an identity but sometimes the '$=$' symbol is used instead.
      E.g. $5(x - 7) \equiv 5x - 35$ is an identity because the LHS $=$ RHS for all values of x
              $(x + 5)(x - 8) \equiv x^2 - 3x - 40$ is an identity because the LHS $=$ RHS for all
                                  values of x
      Hint: Check that the expressions on the LHS and the RHS are the same.

A *formula* is a relationship with an '$=$' symbol where the *letters stand for defined quantities*.
      E.g. $F = ma$ and $v = u + at$ are both formulae.

E.g. 1 Decide whether the following are equations, expressions, identities or formulae:
      (a) $(x + 1)^2 \equiv x^2 + 2x + 1$ (b) $7y + 10$ (c) $V = IR$
      (d) $7x + 11 = x - 9$ (e) $x^2 - 3x + 10$ (f) $A = \pi r^2$
      (g) $x(x + 1) = x^2 + x$ (h) $x^2 - 7x = 0$

      Working: (a) $(x + 1)^2 \equiv x^2 + 2x + 1$ **Identity — expand the brackets and the expression on the LHS is the same as the expression on the RHS.**

Equating coefficients

Equating coefficients is when the numbers in front of x^2, x etc are put equal to each other. For example, if $ax^2 + bx + c \equiv x^2 + 5x - 8$ then:
      Equating coefficients of x^2: $a = 1$
      Equating coefficients of x: $b = 5$
      Equating the constant term: $c = -8$

E.g. 2 Find the values of a, b and c that turn these statements into identities (i.e. true for all values of x):
  (a) $x^2 + ax + b \equiv x^2 + 3x + 2x + 1$
  (b) $ax + b \equiv 2(x + 4) + 7(x + 1)$

  Working: (a) $x^2 + ax + b \equiv x^2 + 3x + 2x + 1$
  The RHS can be simplified to $x^2 + 5x + 1$
  So $x^2 + ax + b \equiv x^2 + 5x + 1$
  Equating coefficients of x: $a = 5$
  Equating the constant term: $b = 1$

E.g. 3 Write the '$=$' or the '$\equiv$' symbol in the box to make each statement mathematically correct.
  (a) $4x - 8 \,\square\, x - 3$ (b) $4x - 8 \,\square\, 2(2x - 4)$

  Working: (a) $4x - 8 \,\square\, x - 3$
  The expression on the LHS is not the same as the one on the RHS so it is only true for specific values. Hence, we need the '$=$' symbol

Explanation: **Expressions, equation, formula and identities**

Solutions to Starter and E.g.s

Exercise
9-1 class textbook: p105 M4.6 Qu 1-8
A*-G class textbook: No exercise
9-1 homework book: p37 M4.6 Qu 1-8
A*-G homework book: No exercise

Summary
An *expression* does not contain an '$=$' or an '$\equiv$' symbol.
An *equation* contains an '$=$' symbol and is *only true for specific values* of the unknown work.
An *identity* is a relationship which is *true for all values* of the unknown. Check that the expressions on the LHS and the RHS are the same.
A *formula* is a relationship with an '$=$' symbol where the *letters stand for defined quantities*.