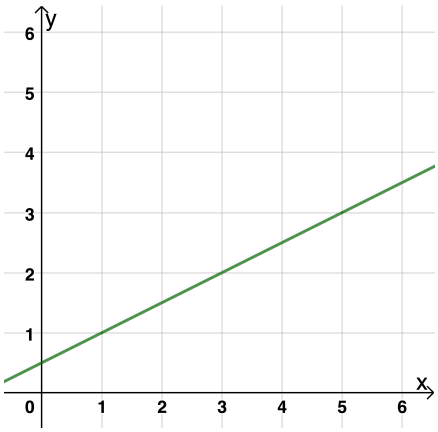
Gradients of parallel and perpendicular lines

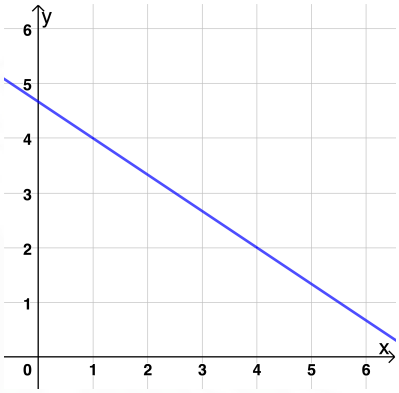
Starter

1. **(Review of last lesson)**
Find the equation of the line passing through the points $(4, -7)$ and $(-1, 8)$.

2. (a) Draw the diagram to the right and find the gradient of the ***green line***.
(b) Draw a line ***parallel*** to the ***green line*** and calculate its gradient. What do you notice?



3. (a) Draw the diagram to the right and find the gradient of the ***blue line***.
(b) Draw a line ***perpendicular*** to the ***blue line*** and calculate its gradient. What do you notice?



Notes

Parallel lines

Lines that are ***parallel*** have the ***same gradient***.

E.g. 1 Write down the equation of two lines that are parallel to:

(a) $y = 2x + 7$ (b) $3x + 8y = 15$ (c) $\frac{y}{x} = 4$

Working: (a) Any line of the form $y = 2x + k$, where k is a number.
E.g. $y = 2x + 5$ and $y = 2x - 39$

E.g. 2 Find the equation of the line which is parallel to the line stated and goes through given point:

(a) $y = 5x - 7$ $(1, 8)$ (b) $y = 7 - 9x$ $(2, -15)$

Working: (a) A line parallel to $y = 5x - 7$ is of the form $y = 5x + k$
Substitute $(1, 8)$: $8 = 5 \times 1 + k$
$8 = 5 + k$
$k = 3$
The equation of the required line is $y = 5x + 3$.

Perpendicular lines

From the starter, it was found that lines with gradients $-\frac{2}{3}$ and $\frac{3}{2}$ are perpendicular. If the gradients are multiplied together, their product is -1

$$-\frac{2}{3}\times\frac{3}{2}=-\frac{6}{6}=-1$$

In fact, for all ***perpendicular*** lines, the ***product of the gradients is*** -1 i.e. $m_1\times m_2=-1$

E.g. 3* A line, L, has gradient $\frac{a}{b}$. Given that the product of perpendicular gradients is -1, find the gradient of the line which is perpendicular to L.

What has happened to $\frac{a}{b}$ to get $-\frac{b}{a}$?

The fraction has flipped and it has changed sign. The technical name is ***negative reciprocal*** i.e. $-\frac{b}{a}$ is the ***negative reciprocal*** of $\frac{a}{b}$.

The reciprocal of a number is "1 over the number"

E.g. $\frac{1}{6}$ is the reciprocal of 6 and vice versa.

The reciprocal of $\frac{a}{b}$ is $\frac{1}{\frac{a}{b}}=1\div\frac{a}{b}=1\times\frac{b}{a}=\frac{b}{a}$

To find a perpendicular gradient, find the ***negative reciprocal*** i.e. "flip it and change the sign".

E.g. 4 Copy and complete the table of perpendicular gradients:

Gradient	Perpendicular gradient
5	
-8	
$\frac{1}{7}$	
$-\frac{4}{5}$	
$1\frac{3}{4}$	
-2.5	

Working:

Gradient	Perpendicular gradient
5	$-\frac{1}{5}$

E.g. 5 Write down the equation of two lines that are perpendicular to:

(a) $y = 3x - 8$ (b) $2x + y = 9$ (c) $5x + 7y = 23$

Working: (a) The gradient of $y = 3x - 8$ is 3.

The gradient perpendicular to 3 is $-\frac{1}{3}$ ***negative reciprocal***

Any line of the form $y = -\frac{1}{3}x + k$, where k is a number.

E.g. $y = -\frac{1}{3}x + 1$ and $y = -\frac{1}{3}x - 65$

E.g. 6 Find the equation of the line perpendicular to the one given and passing through the point stated:

(a) $y = -6x + 1$ $(12, -3)$

(b) $y = \frac{4}{3}x + 15$ $(-8, 5)$

(c) $3x + 7y = 1$ $(-6, 2)$

Working: (a) The gradient of $y = -6x + 1$ is -6.

The gradient perpendicular to -6 is $\frac{1}{6}$ ***negative reciprocal***

The equation of the line is of the form $y = \frac{1}{6}x + c$.

Substituting $(12, -3)$: $-3 = \frac{1}{6} \times 12 + c$

$-3 = 2 + c$

$c = -5$

The equation of the line is $y = \frac{1}{6}x - 5$.

Video: Parallel lines

Video: Perpendicular lines

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p197 M6.14 Qu 26, 37, 38
	p200 E6.8 Qu 1ace…, 2ace, 4ace…, 5-11
A*-G class textbook:	p184 E6.6 Qu 1-6
9-1 homework book:	p70 M6.14 Qu 1-10; p72 E6.8 Qu 1-10
A*-G homework book:	p54 E6.6 Qu 1-7

Summary

Lines that are ***parallel*** have the ***same gradient***.

To find a perpendicular gradient, find the ***negative reciprocal*** i.e. “flip it and change the sign”.