Linear inequalities

Starter

1. **(Review of last lesson)**
When a number is doubled and then added to 67, the result is 213. Find the number.

2. **(Review of last lesson)**
Two angles of an isosceles triangle are a and $a + 10$. Find two possible values of a.

Inequality notation:

$<$ less than	$>$ more than
$\leq$ less than or equal to	$\geq$ more than or equal to

N.B. The inequality sign points to the smaller number

3. (a) Solve the equation $2x - 1 = 5$.
(b) Using similar working to part (a), find the range of values of x that satisfy $2x - 1 \geq 5$.

Notes

Solving inequalities is similar to solving equations apart from one important difference.

E.g. 1 (a) Solve the equation $7 - 2x = 11$.
(b) Find the range of values of x that satisfy $7 - 2x < 11$ using the following methods:
(i) by moving the the $-2x$ to the other side.
(ii) by keeping the $-2x$ on the left-hand side.

Multiplying/dividing by negative numbers

We know that $3 < 4$. If we multiply both numbers by -1 we get -3 and -4 but what is the inequality symbol in place of the box $-3 \square -4$.

$$3 < 4 \qquad\qquad -3 > -4$$

The inequality symbol has 'changed direction'.

With ***inequalities***, when ***multiplying*** or ***dividing*** by a ***negative number***, ***change*** the ***direction*** of the ***inequality symbol***.

E.g. 2 Without moving the x, find the range of values of x that satisfy:

(a) $-2x < 8$ (b) $-\frac{x}{3} > -5$

Working: (a)

$$-2x < 8$$

Divide by -2, change direction of inequality:

$$x > \frac{8}{-2}$$

$$x > -4$$

Single inequalities

In single inequalities, the variable (i.e. the letter) is written on the left-hand side.

$x >$ a number $x \leq$ a number

Inequalities diagram

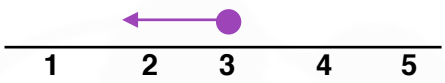
Inequalities can be shown on a number line but we must distinguish between inequalities that include the number, $\leq$ and $\geq$, and those that don't include the number, $<$ and $>$.

- For $\leq$ and $\geq$: use a ball ●
- For $<$ and $>$: use a circle ○

E.g. 3 Express these inequalities diagrammatically:

(a) $x \leq 3$ (b) $x > 2$ (c) $2 < x \leq 3$

Working: (a)



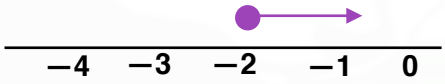
E.g. 4 Solve these inequalities, expressing your answer as both an inequality and is diagram form:

(a) $-3x - 7 \leq -1$ (b) $65 - 7x < -12$

Working: (a) $-3x - 7 \leq -1$

Add 7: $-3x \leq 6$

Divide by -2: $x \geq -2$ ***direction of inequality changes***



Compound inequalities

A ***compound inequality*** combines two inequalities into one.

For example, $1 < x \leq 5$ or $-4 \leq h \leq 2$.

In a compound inequality:

- The inequality signs must both ***point to the left*** (e.g. $< x <$)
- The ***smaller number*** is on the ***left***
- The ***bigger number*** is on the ***right***

Smaller number $\leq x \leq$ Bigger number

For example, $3 < x \leq 9$ means that $x > 3$ and $x \leq 9$.

Success Criteria — solving compound inequalities

1. Write down two ***separate*** inequalities
2. ***Solve*** them ***separately***
3. ***Combine*** the inequalities back into one

E.g. 5 Solve the inequalities:

(a) $8 < 3x - 10 < 23$ (b) $17 < 6x + 5 \leq 29$

(c) $7 < 15 - 4x < 39$ (d) $x - 19 \leq 5x - 3 < x + 5$

Working: (a) $8 < 3x - 10 < 23$

$8 < 3x - 10$ $\quad$ $3x - 10 < 23$ ***2 separate inequalities***

$18 < 3x$ $\quad$ $3x < 33$ ***solve separately***

$6 < x$ $\quad$ $x < 11$

Combine into 1 inequality: $6 < x < 11$

Video: Solving inequalities with one sign
Video: Solving inequalities (two signs)

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p509 M16.1 Qu 1-7 (may need to explain Qu 5)
A*-G class textbook:	p465 M16.1 Qu 1-6 (may need to explain Qu 4)
9-1 homework book:	p172 M16.1 Qu 1-4
A*-G homework book:	p128 M16.1 Qu 1-6

Summary

With ***inequalities***, when ***multiplying*** or dividing by a ***negative number***, ***change*** the ***direction*** of the ***inequality sign***.

Single inequalities: $x >$ ***a number*** $x \leq$ ***a number***

Inequalities diagrams: For $\leq$ and $\geq$: use a ball ●
For $<$ and $>$: use a circle ○

Compound inequality: **Smaller number** $\leq x \leq$ **Bigger number**

Solving compound inequalities:

1. Write down two ***separate*** inequalities
2. ***Solve*** them ***separately***
3. ***Combine*** the inequalities back into one