

Negative indices

Starter

1. **(Review of last lesson)**
Simplify: (a) 125^0 (b) $8x^6 \times 5x^9$ (c) $(a^3)^4 \div a^9$

2. **(Review of last lesson)**
Find the value of x in the following expressions:
(a) $a^x \div a^7 = a^9$ (b) $(6^{3x})^4 = 6^{48}$

2. With a partner, decide whether the answer of the following is A, B, C or D. Justify your answer with working or an explanation.

2^{-3} **A.** -6 **B.** -8 **C.** $\frac{1}{8}$ **D.** -1

Notes

Negative indices (“one over...”):

$$a^{-1} = \frac{1}{a}$$

$$a^{-n} = \frac{1}{a^n}$$

This leads to $\frac{1}{a^{-1}} = a$

The word **reciprocal** is often associated with negative indices because the reciprocal of 5 is $\frac{1}{5}$.
The reciprocal of a number or expression is “one over” the number or expression.

Integers raised to negative powers

E.g. 1 Without using a calculator, simplify the following:

(a) 7^{-1} (b) $\frac{1}{9^{-1}}$ (c) 5^{-3} (d) $\frac{1}{6^{-2}}$

Working: (a) $7^{-1} = \frac{1}{7}$

Exercise

9-1 class textbook: p42 M2.7 Qu 1, 2ace..., 3-6
A*-G class textbook: p39 E2.1 Qu 1, 2ace..., 3-5
9-1 homework book: p13 M2.7 Qu 1-4
A*-G homework book: p10 E2.1 Qu 1-4

Fractions raised to negative powers

E.g. 2 By using division by a fraction, and without a calculator, state the value of:

(a) $\left(\frac{1}{4}\right)^{-1}$

(b) $\left(\frac{2}{3}\right)^{-1}$

Working:

(a) $\left(\frac{1}{4}\right)^{-1} = \frac{1}{\frac{1}{4}}$ *negative indices \equiv "one over..."*
 $= 1 \div \frac{1}{4}$ *dividing by fraction*
 $= 1 \times \frac{4}{1}$ *flip fraction and multiply*
 $= 4$

When a **fraction** is raised to a **negative power**, "**flip**" the fraction and **make the power positive**.

i.e. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

E.g. 3 Without a calculator, find the value of:

(a) $\left(\frac{1}{12}\right)^{-1}$

(b) $\left(\frac{7}{8}\right)^{-1}$

(c) $\left(\frac{1}{3}\right)^{-2}$

(d) $\left(\frac{3}{5}\right)^{-2}$

Working: (a) $\left(\frac{1}{12}\right)^{-1} = \frac{12}{1} = 12$

(b) $\left(\frac{7}{8}\right)^{-1} = \frac{8}{7}$

(c) $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^2 = 3^2 = 9$

(d) $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$

N.B. If a mixed number is raised to a negative power, **turn it into an improper fraction** and then find the negative reciprocal.

E.g. 4 Without a calculator, find the value of:

(a) $\left(1\frac{1}{5}\right)^{-3}$

(b) $\left(2\frac{1}{4}\right)^{-2}$

Working: (a) $\left(1\frac{1}{5}\right)^{-3} = \left(\frac{6}{5}\right)^{-3} = \left(\frac{5}{6}\right)^3 = \frac{5^3}{6^3} = \frac{125}{216}$

Exercise

9-1 class textbook: p43 M2.7 Qu 7,8
A*-G class textbook: p40 M2.7 Qu 6, 7
9-1 homework book: p13 M2.7 Qu 5-7
A*-G homework book: p11 E2.1 Qu 5, 7

Negative Indices involving algebra

E.g. 5 Simplify: (a) $\frac{1}{a^{10}} \times a^7$ (b) $g^6 \div g^{-6}$ (c) $(b^{-5})^{-1}$

Working: (a) $\frac{1}{a^{10}} \times a^7 = a^{-10} \times a^7 = a^{-3} = \frac{1}{a^3}$

Video: [Negative indices](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p42 M2.7 Qu 9-14
A*-G class textbook: p37 E2.1 Qu 8-10
9-1 homework book: p13 M2.7 Qu 8-10
A*-G homework book: p10 E2.1 Qu 6

Summary

Negative indices ("one over..."): $a^{-1} = \frac{1}{a}$ $a^{-n} = \frac{1}{a^n}$ $\frac{1}{a^{-1}} = a$

The word **reciprocal** is often associated with negative indices because the reciprocal of 5 is $\frac{1}{5}$.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

If a mixed number is raised to a negative power, **turn it into an improper fraction** and then find the negative reciprocal.