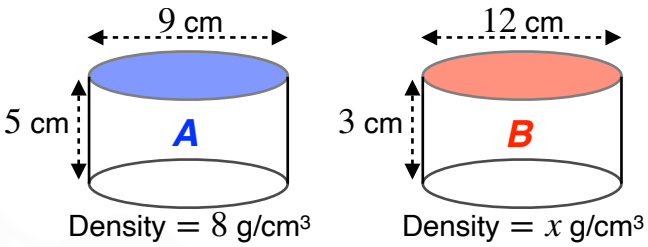
Pythagoras' Theorem

Starter

1. **(Review of last lesson)**
The two solid cylinders shown have the same mass. Calculate the density, x g/cm^3, of cylinder B.

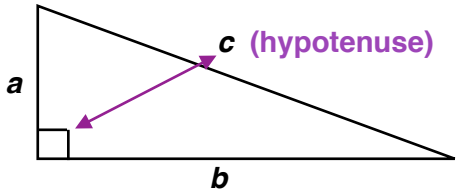


Notes

You met Pythagoras' Theorem last academic year:

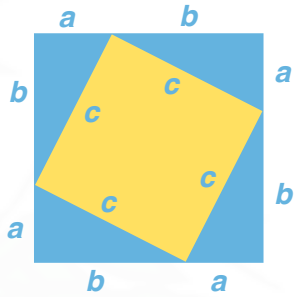
$$a^2 + b^2 = c^2$$

where c is the length of the hypotenuse of a right-angled triangle and a and b are the lengths of the two shorter sides.



N.B. The hypotenuse is the longest side. It is opposite the right-angle.

E.g. 1 By finding the area of the large square in two different ways, prove Pythagoras Theorem i.e. that $a^2 + b^2 = c^2$



Working: Length of one side of big square $= a + b$

Area of large square $= (a+b)^2$

$$= (a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

Also, area of large square = area of small square + area of 4 triangles

$$= c^2 + 4 \times \left(\frac{a \times b}{2} \right)$$
$$= c^2 + 2ab$$

Equating the two formulae for the area of the big area:

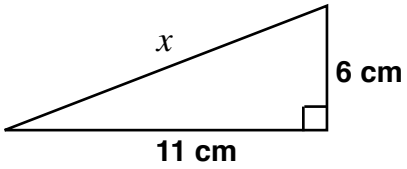
$$a^2 + 2ab + b^2 = c^2 + 2ab$$

Subtracting $2ab$ from both sides: $a^2 + b^2 = c^2$

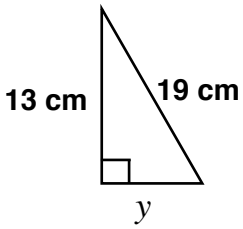
N.B. Always label the hypotenuse — this value is on its own on one side of the formula.

E.g. 2 Find the length of the missing side of these right-angled triangles:

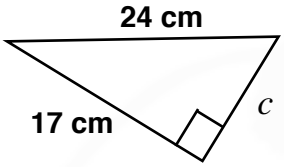
(a)



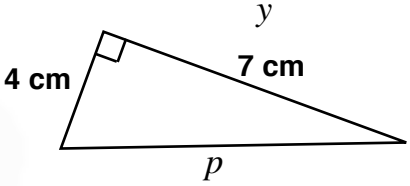
(b)



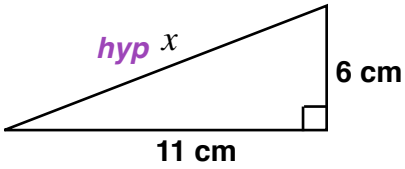
(c)



(d)



Working: (a) ***Label the hypotenuse***



$$a^2 + b^2 = c^2$$
$$6^2 + 11^2 = x^2$$
$$36 + 121 = x^2$$
$$x^2 = 157$$
$$x = \sqrt{157} = 12.5 \text{ (3 s.f.)}$$

E.g. 3 Find the distance between the points $(-3, 7)$ and $(8, -1)$.

E.g. 4 Determine whether a triangle with sides 19 cm, 16 cm, 10 cm has a right angle.

Working: If the triangle has a right-angle the lengths will satisfy Pythagoras' theorem.

$a^2 + b^2 = c^2$: LHS: $10^2 + 16^2 = 100 + 256 = 356$

RHS: $19^2 = 361$

Since $356 \neq 361$, the lengths do not satisfy Pythagoras' theorem and so the triangle does not have a right angle.

E.g. 5 The lengths in a right-angled triangle are x, $3x$ and 45 with 45 cm being the hypotenuse. Find x.

Video: Pythagoras' Theorem

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p315 M10.5 Qu 1-8; p317 M10.6 Qu 1-18
A*-G class textbook:	p278 M10.5 Qu 1-6; p280 M10.6 Qu 1-20 even
9-1 homework book:	p107 M10.5 Qu 1-8; p108 M10.6 Qu 1-12
A*-G homework book:	p78 M10.5 Qu 1-5; p79 M10.6 Qu 1-12

Summary

Pythagoras' Theorem: $a^2 + b^2 = c^2$

The hypotenuse is the longest side.
It is opposite the right-angle.