Rearranging formulae (including 2 steps, brackets and simple fractions)

Starter

1. **(Review of last lesson)**
   Solve the inequalities, expressing your answer in algebraic and diagrammatic form:
   (a) $4x - 3 < 25$ (b) $-3x + 4 \leq 25$

2.   Solve the inequality $7 < 17 - 5x < 32$.

Notes

A formula has an $=$ symbol and at least two letters in it. One letter is usually on its own on the left hand side. This letter is called the *subject of the formula*. The subject of the formula must always be positive.

Changing the subject a formula means *rearranging the formula* so that another letter is on its own on the LHS.

Do not copy

For example, consider the formula $v = u + at$.   The subject of the formula is v.

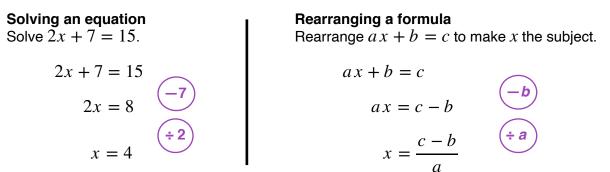We could rearrange to make u the subject of the formula $v = u + at$.

$$v = u + at \qquad \text{\textit{u is positive so we don't want to move it to the other side}}$$

$$v - at = u \qquad \boxed{-at} \quad \text{\textit{u is positive, on its own and at the top but it is not on the LHS}}$$

$$u = v - at \qquad \text{\textit{New subject on the LHS}}$$

Or we could rearrange to make a the subject of the formula.

$$v = u + at \qquad \text{\textit{a is positive so we don't want to move it to the other side}}$$

$$v - u = at \qquad \boxed{-u} \quad \text{\textit{a is not on its own so we need to move the t}}$$

$$\frac{v - u}{t} = a \qquad \boxed{\div t} \quad \text{\textit{Divide by t since division is the opposite to multiplication}}$$

$$a = \frac{v - u}{t} \qquad \text{\textit{New subject on the LHS}}$$

Please start copying again.

The principles behind solving an equation are exactly the same when you are changing the subject of a formula so we use **SABMIB** to decide which operation to do at each stage.

Solving an equation
Solve $2x + 7 = 15$.

$$2x + 7 = 15$$
$$2x = 8 \qquad \boxed{-7}$$
$$x = 4 \qquad \boxed{\div 2}$$

Rearranging a formula
Rearrange $ax + b = c$ to make x the subject.

$$ax + b = c$$
$$ax = c - b \qquad \boxed{-b}$$
$$x = \frac{c - b}{a} \qquad \boxed{\div a}$$

When rearranging an equation we need to make sure **PLOT** is in place at the end:

Positive — the new subject must be positive

LHS — the new subject must be on the LHS (if you have $v - at = u$ write $u = v - at$)

On its own — no other letters or number must be with the new subject

Top — the new subject must not be in the *denominator* of a fraction

> ***N.B.***   LHS ≡ Left-hand side
> "Don't lose the **PLOT**."

E.g. 1 Rearrange the formula to make x the subject:

(a) $5x + y = z$ (b) $y = px - q$ (c) $a - b = b + cx$

Working: (a)   *x is positive so we don't want to move it to the other side*
Addition before multiplication $5x + y = z$
Subtract y from both sides $5x = z - y$
Divide both sides by 5 $x = \dfrac{z - y}{5z}$

E.g. 2 Rearrange the formula to make x the subject:

(a) $\dfrac{x}{p} = q - r$ (b) $\dfrac{x}{a} - 7 = b$ (c) $q = \dfrac{x}{a} - p$

Working: (a)   *x is positive so we don't want to move it to the other side*
$$\frac{x}{p} = q - r$$
Multiply both sides by p $x = p(q - r)$

N.B.   ***Expand*** brackets ***before rearranging unless the new subject is in front of the bracket***

E.g. 3 Rearrange the formula to make x the subject of the formula:

(a) $a(x - b) = c$ (b) $a = t(4x + y)$ (c) $x(a - b) = c$

Working: (a)   *x is positive so we don't want to move it to the other side*
$$a(x - b) = c$$
x is not in front of the brackets so expand $ax - ab = c$
Subtraction before multiplication
Add ab from both sides $ax = c + ab$
Divide both sides by a $x = \dfrac{c + ab}{a}$

N.B.   When the new subject is negative, it is usually a good idea to ***prioritise making it positive***

E.g. 4 Rearrange the formula to make y the subject of the formula:

(a) $a = b - cy$ (b) $a^2 = b^2 - cy$ (c) $a - 7y = b$

Working: (a)   *y is negative so we need to move it to the other side*
$$a = b - cy$$
Add cy to both sides $a + cy = b$
Subtraction before multiplication
Subtract a from both sides $cy = b - a$
Divide both sides by c $y = \dfrac{b - a}{c}$

Exercise

9-1 class textbook:	p165 M6.5 Qu 6-13
A*-G class textbook:	p157 M6.6 Qu 6-14
9-1 homework book:	p58 M6.5 Qu 1-10
A*-G homework book:	p44 M6.6 Qu 1-10

Summary
Changing the subject a formula means *rearranging the formula* so that another letter is on its own on the LHS.

The principles behind solving an equation are exactly the same when you are changing the subject of a formula so we use **SABMIB** to decide which operation to do at each stage.

When rearranging an equation we need to make sure **PLOT** is in place at the end:

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Top — the new subject must not be in the denominator of a fraction