

Repeated percentage increase or decrease

Starter

1. **(Review of last lesson)** When VAT is added to the cost of a tyre, its price increases from £54.50 to £61.04. What is the rate at which VAT is charged?
2. **(Review of last lesson)** Two years ago a new house was worth £270000. In each year since it was built, its value increased by 6%. How much is the house now worth?

Notes

In question 2 from the starter, the answer could have been found quicker by doing one calculation rather than two.

$$\text{Value of house after 2 years} = 270000 \times 1.06 \times 1.06 = 270000 \times 1.06^2 = 303372$$

The **power** refers to the **number of time periods** (in this case, years) at that percentage rate.

Since the percentage rate remained constant, powers can be used.

E.g. 1 A new house is worth £120000. Write down the calculation to find its value:

- (a) after 4 years when its value increased by 5% each year
- (b) after 7 years when its percentage increase was 2.6% a year
- (c) after 3 years when its value **decreased** by 2% each year
- (d) increase by 3% in the first 2 years and then increased by 1.7% in the next 6 years.

Working: (a) Increase by 5% \Rightarrow $\times 1.05$
Number of years at 5% is 4 so the power of 1.05 is 4.
New value of house = 120000×1.05^4

Compound interest

A common example of repeated percentage increase is the **compound interest** paid on savings by banks. Banks pay you money, called interest, when you open a bank account. Interest earned is based on how much you have in the bank.

E.g. 2 A bank pays 2% interest on savings so long as the money is untouched for 4 years. Sara invests £5000 in the bank.

- (a) How much money does she have in the bank after 4 years?
- (b) How much interest has she earned?

Working: (a) Increase by 2% for 4 years \Rightarrow $\times 1.02^4$
Amount in bank after 4 years = $5000 \times 1.02^4 = £5412.16$

With **compound interest**, you earn "**interest on the interest**", meaning that the interest you earn one year is included in the calculation of the interest the following year.

Compound interest vs. simple interest

Another type of interest is called **simple interest** and as the same suggest is a simpler calculation. With simple interest, the interest earned in the first year is simply multiplied by the number of years.

E.g. £2500 is invested at 8 % interest. Calculate the value of the investment and how much interest was earned after 3 years for (a) **compound interest** and (b) **simple interest**.

Working:

(a) **Compound interest**

Increase by 8 % for 3 years $\Rightarrow \times 1.08^3$

Amount in bank after 3 years = 2500×1.08^3
= £3149.28

Interest earned = $3149.28 - 2500$
= £649.28

(b) **Simple interest**

8 % of £2500 $\Rightarrow 0.08 \times 2500$

Interest gained in 1 year \equiv find 8 % of £2500
= 0.08×2500
= £200

Interest gained in 3 years = 200×3
= £600

Amount in bank after 3 years = $2500 + 600$
= £3100

N.B. **Compound interest** — finds the total amount first

Simple interest — finds the amount of interest first

Once period of time is longer than 1 year, compound interest earns more interest than simple interest.

E.g. 3 Jack invests £9000 at 3 % interest. Calculate the value of the investment and how much interest was earned after 5 years for (a) compound interest and (b) simple interest.

Working: (a) **Compound interest:** Increase by 3 % for 5 years $\Rightarrow \times 1.03^5$
Amount in bank after 3 years = $9000 \times 1.03^5 = 10433.47$
Interest earned = $10433.47 - 9000 = £1433.47$

Repeated percentage increase and decrease has lots of applications.

E.g. 4 The population of Ripon is currently 16000 inhabitants. If the population increases by 1 % per year, what would the population be:

(a) 3 years from now (b) 5 years **ago**.

Round your answers to the nearest number of people.

E.g. 5 At the start there are 3000 bacteria in a petri dish. They decrease by 6 % per hour.

(a) How many bacteria will there be in 8 hours?

(b) After how many complete hours will the bacteria have dropped to less than half the original bacteria?

Working: (a) Decrease by 6 % $\Rightarrow \times 0.94$
Bacteria after 8 hours = $3000 \times 0.94^8 = 1829$ bacteria

Video:

[Repeated percentage increase or decrease](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook: p31 M2.3 Qu 1-15

A*-G class textbook: p29 M2.3 Qu 1-15

9-1 homework book: p9 M2.3 Qu 1-8

A*-G homework book: p7 M2.3 Qu 1-8

Summary

With **compound interest**, you earn “**interest on the interest**”, meaning that the interest you earn one year is included in the calculation of the interest the following year.

The **power** refers to the **number of time periods** (in this case, years) at that percentage rate.

With **simple interest**, the interest earned in the first year is simply multiplied by the number of years.

N.B. **Compound interest** — finds the total amount first
Simple interest — finds the amount of interest first

