Simplifying surds

Starter

1. **(Review of last lesson)** Find the acute angle between the two diagonals of a rectangle whose sides are 6 cm and 9 cm.

2. (a) Write down the first ten square numbers.
   (b) Hence simplify: (i) $\sqrt{36}$ (ii) $\sqrt{81}$ (iii) $\sqrt{32}$

Notes

A surd includes the square root of a non-square number.

E.g. $\sqrt{2}$ and $\sqrt{45}$ are surds but $\sqrt{49}$ is not since $\sqrt{49} = 7$

Surds are ***irrational*** numbers so cannot be written as fractions i.e. $\sqrt{2} = 1.4142135\ldots$

It is more accurate to leave an answer in surd form than write it in decimal form. This is especially useful when the number could be used in a further calculation.

Simplifying surds

To simplify a surd, find the ***largest square number that is a factor***.

E.g. Simplify: (a) $\sqrt{20}$ (b) $\sqrt{32}$

Working: (a) $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

E.g. 1 Without a calculator, simplify these surds:

(a) $\sqrt{12}$ (b) $\sqrt{50}$ (c) $\sqrt{48}$ (d) $\sqrt{200}$

Working: (a) $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Multiplying/dividing surds

Working backwards from what is written above: $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

When two surds are multiplied or divided:

1. Rewrite the calculation as a single surd by multiplying or dividing the numbers.
2. Simplify the resulting surd where possible.

N.B. If, when multiplying, the product of the two surds is large, simplify the individual surds before multiplying:

$$\begin{aligned}\sqrt{75} \times \sqrt{8} &= \sqrt{25 \times 3} \times \sqrt{4 \times 2} \\ &= 5\sqrt{3} \times 2\sqrt{2} \\ &= 10\sqrt{6}\end{aligned}$$

E.g. 2 Without a calculator, simplify: (a) $\sqrt{8} \times \sqrt{2}$ (b) $\dfrac{\sqrt{27}}{\sqrt{12}}$

(c) $\sqrt{15} \times \sqrt{3}$ (d) $\dfrac{\sqrt{240}}{\sqrt{6}}$

Working: (a) $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$

Adding and subtracting surds

E.g. 3 Given that $2x + 3x = 5x$, write down, in surd form, the value of $2\sqrt{7} + 3\sqrt{7}$.

Working: $2\sqrt{7} + 3\sqrt{7} = 5\sqrt{7}$

Before adding or subtracting surds make sure the surd is the same.

$3\sqrt{5} + 8\sqrt{7}$ cannot be added since the surds are different.

At first glance $7\sqrt{24} + 5\sqrt{6}$ looks like it cannot be added but $\sqrt{24}$ can be expressed in the form $k\sqrt{6}$:

$$\begin{aligned} 7\sqrt{24} + 5\sqrt{6} &= 7\sqrt{4 \times 6} + 5\sqrt{6} \\ &= 7\sqrt{4} \times \sqrt{6} + 5\sqrt{6} \\ &= 7 \times 2 \times \sqrt{6} + 5\sqrt{6} \\ &= 14\sqrt{6} + 5\sqrt{6} \\ &= 19\sqrt{6} \end{aligned}$$

E.g. 4 Without a calculator, simplify, where possible:

(a) $7\sqrt{5} - 3\sqrt{5}$ (b) $\sqrt{32} + 3\sqrt{2}$

(c) $2\sqrt{27} + 6\sqrt{18}$ (d) $5\sqrt{27} + 3\sqrt{28}$

Working: (a) $7\sqrt{5} - 3\sqrt{5} = 4\sqrt{5}$

Video: Surds

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p12 E1.2 Qu 1ace…, 2ace…, 3-5, 6ace…, 7, 8ace…, 9
A*-G class textbook:	p12 E1.2 Qu Qu 1ace…, 2ace…, 3, 4, 5ace…, 6, 7ace…,
9-1 homework book:	p4 E1.2 Qu 1ace…, 2, 3ace…, 4ace…, 5, 6
A*-G homework book:	p4 E1.2 Qu 1ace…, 2ace…, 3, 4ace…

Summary

To simplify a surd, find the ***largest square number that is a factor***.

Multiplying/dividing surds:

1. Rewrite the calculation as a single surd by multiplying/dividing the numbers.
2. Simplify the resulting surd where possible.

Before adding or subtracting surds make sure the surd is the same.