

Area and Volume Problems, including surface area of a cylinder

**Starter**

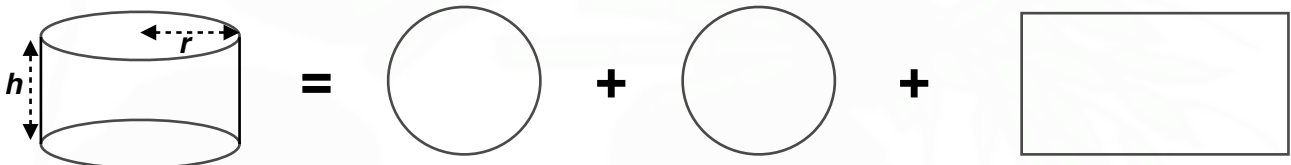
1. **(Review of last lesson)** The volume of a cylinder is  $260 \text{ cm}^3$ . Given that its height is  $11 \text{ cm}$ , calculate its diameter to 3 s.f..

**Working:**  $V = 260, h = 11$   
 Volume,  $V = \pi r^2 h$ :  $260 = \pi \times r^2 \times 11$   
 $260 = 11\pi r^2$   
 $\frac{260}{11\pi} = r^2$   
 $r = \sqrt{\frac{260}{11\pi}}$   
 $r \approx 2.7429$   
 The diameter of the cylinder is  $5.49 \text{ cm}$  (3 s.f.)

2. **(Review of last lesson)** A solid cylinder of radius  $10 \text{ cm}$  and length  $14 \text{ cm}$  is melted down and recast into a solid cube. Find the length of the side of the cube.

**Working:**  $r = 10, h = 14$   
 Volume,  $V = \pi r^2 h$ :  $V = \pi \times 10^2 \times 14 = 1400\pi$   
 The length of the side of the cube is  $\sqrt[3]{1400\pi} = 16.4 \text{ cm}$ .

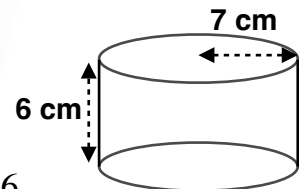
3. Using the diagram below, find a formula for the surface area of a cylinder.



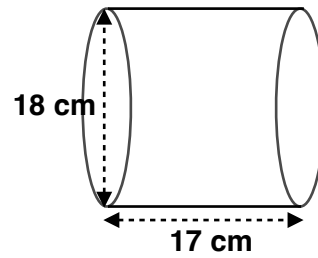
**Working:** The length of the rectangle is equal to the circumference of the circle.  
 Surface area = Area of circle + Area of circle + Area of rectangle  
 $= \pi r^2 + \pi r^2 + 2\pi r \times h$   
 $= 2\pi r^2 + 2\pi rh$

4. Calculate the surface area of the cylinder.  
 Give your answer to 3 s.f..

**Working:**  $r = 7$  and  $h = 6$   
 $SA = 2\pi r^2 + 2\pi rh$ :  $SA = 2\pi \times 7^2 + 2\pi \times 7 \times 6$   
 $= 98\pi + 84\pi$   
 $= 572 \text{ cm}^2$  (3 s.f.)



**E.g. 1** Calculate the surface area of the cylinder.  
Give your answer in terms of  $\pi$ .



**Working:**  $r = 9$  and  $h = 17$   
 $SA = 2\pi r^2 + 2\pi rh:$   $SA = 2\pi \times 9^2 + 2\pi \times 9 \times 17$   
 $= 162\pi + 306\pi$   
 $= 468\pi \text{ cm}^2$

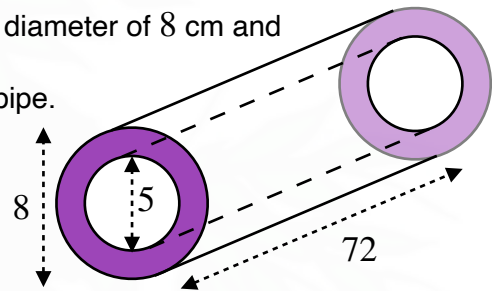
**E.g. 2** Calculate the height of a cylinder whose surface area is  $124\pi$  given that the radius of the cross-section is 6 cm.

**Working:**  $SA = 2\pi r^2 + 2\pi rh:$   $2\pi \times 6^2 + 2\pi \times 6h = 124\pi$   
**Simplifying:**  $72\pi + 12\pi h = 124\pi$   
**Subtract  $72\pi$ :**  $12\pi h = 52\pi$   
**Divide by  $10\pi$ :**  $h = \frac{52\pi}{12\pi} = \frac{13}{3} \text{ cm}$

**E.g. 3** A cylindrical metal pipe, of length 72 cm, has external diameter of 8 cm and internal diameter of 5 cm.

- (a) Calculate the volume of metal in the length of pipe.  
 (b) Given that the density of the metal is  $6 \text{ g/cm}^3$ , find the mass of the pipe.

Give your answers exactly and include units.



**Working:** (a) The volume of the metal can be found by taking the volume of the inner cylinder away from the outer cylinder.  
 Outer:  $r = 4, h = 72$  Inner:  $r = 2.5, h = 72$   
 Volume,  $V = \pi r^2 h:$   $V = \pi \times 4^2 \times 72 - \pi \times 2.5^2 \times 72$   
 $= 702\pi$

The volume of metal in the length of pipe is  $702\pi \text{ cm}^3$ .

(b) Density =  $\frac{\text{Mass}}{\text{Volume}}$ :  $6 = \frac{\text{Mass}}{702\pi}$   
 Mass =  $6 \times 702\pi = 4212\pi$

The mass of the pipe is  $4212\pi$  grams.

**E.g. 4** Water flows through a circular pipe of internal diameter 3 cm at a speed of 16 cm/s. If the pipe is full, how many litres of water issue from the pipe in one minute?

**Working:**  $r = 1.5, h = 16$   
 Volume in 1 second,  $V = \pi r^2 h:$   $V = \pi \times 1.5^2 \times 16 = 36\pi$   
 Volume in 1 minute:  $36\pi \times 60 = 6785.8 \text{ cm}^3$   
 Volume of water issue from the pipe in one minute is 6.79 litres.

**Exercise**

9-1 class textbook: p445 M13.8 Qu 1a, 2c, 3, 6  
p439 M13.6 Qu 1, 2, 4-8 (3 needs trigonometry)

A\*-G class textbook: p400 M13.8 Qu 1a, 2c, 3, 7  
p392 M13.2 Qu 13-22

9-1 homework book: p153 M13.8 Qu 1c  
p149 M13.5/13.6 Qu 7-12

A\*-G homework book: p112 E13.5 Qu 1c  
p110 M13.2 Qu 6-10