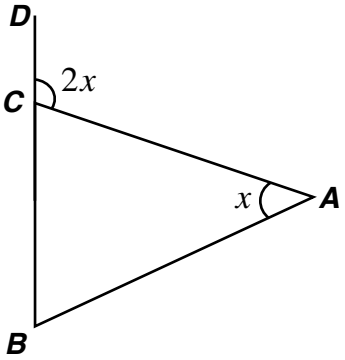
Areas

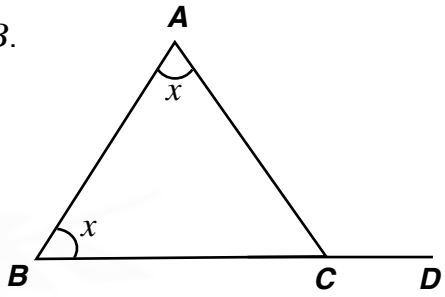
Starter

1. **(Review of last lesson)** Prove that $\triangle ABC$ is isosceles.



Working: $A\hat{C}B = 180° - 2x$ since angles on a straight line add up to $180°$.
$A\hat{B}C + A\hat{C}B + B\hat{A}C = 180°$ since angles in a triangle add up to $180°$.
$A\hat{B}C + 180° - 2x + x = 180°$
$A\hat{B}C = x$
Since $A\hat{B}C = B\hat{A}C = x$ ***and*** $A\hat{C}B \neq x$ the $\triangle ABC$ is isosceles

2. **(Review of last lesson)** $A\hat{C}D$ is double the size of $A\hat{C}B$. Prove that the $\triangle ABC$ is equilateral.



Working: $A\hat{C}D + A\hat{C}B = 180°$ since angles on a straight line add up to $180°$.
But $A\hat{C}D = 2 \times A\hat{C}B$
So $2 \times A\hat{C}B + A\hat{C}B = 180°$
$3 \times A\hat{C}B = 180°$ $\therefore A\hat{C}B = 60°$
$A\hat{C}B + x + x = 180°$ since angles in a triangle add up to $180°$.
$60° + 2x = 180°$ $\Rightarrow$ $2x = 120°$ $\Rightarrow$ $x = 60°$
Since all angles in the triangles are equal to $60°$, the $\triangle ABC$ is equilateral.

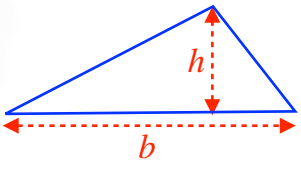
Notes

E.g. 1 Write down the formulae, with accompanying diagrams, for the:
(a) area of a triangle
(b) area of a circle
(c) area of trapezium.

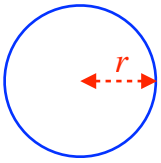
Working: (a) Area of triangle = half base times perpendicular height.

...or... Area of triangle $= \frac{1}{2} b \times h$

...or... Area of triangle $= \frac{b \times h}{2}$

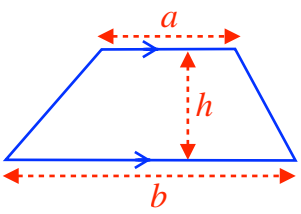


(b) Area of circle $= \pi r^2$



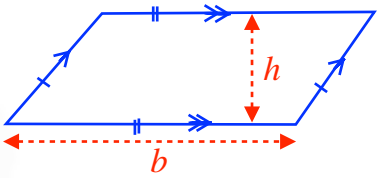
(c) Area of trapezium = half sum of parallel sides times distance between them.

...or... Area of trapezium $= \frac{1}{2}(a+b)h$

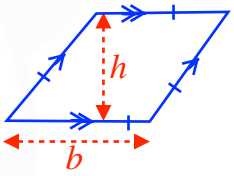


Here are some additional area formulae:

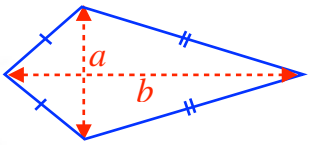
Area of parallelogram = base times perpendicular height
$= b \times h$



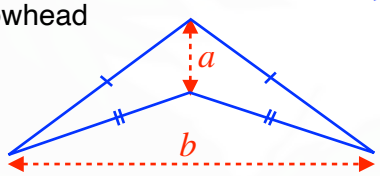
Area of rhombus = base times perpendicular height.
$= b \times h$



E.g. 2 (a) By splitting the area of the kite shown into triangles, find a formula for its area in terms of a and b.



(b) Hence write down a formula for an arrowhead



Working: (a) The horizontal dotted line splits the kite into two equal triangles

Area of each triangle $= \frac{1}{2}b \times \frac{1}{2}a = \frac{1}{4}ab$

Area of kite $= 2 \times \frac{1}{4}ab = \frac{1}{2}ab$

Area of kite = half the product of the diagonals.

$$= \frac{1}{2}a \times b$$

$$= \frac{a \times b}{2}$$

(b) Area of arrowhead = half the product of the diagonals.

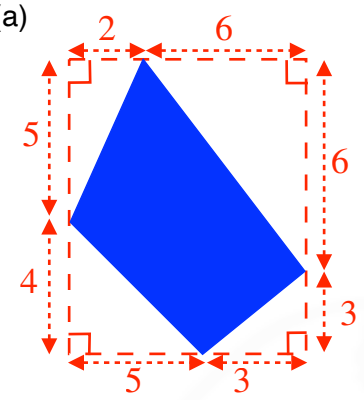
$$= \frac{1}{2}a \times b$$

$$= \frac{a \times b}{2}$$

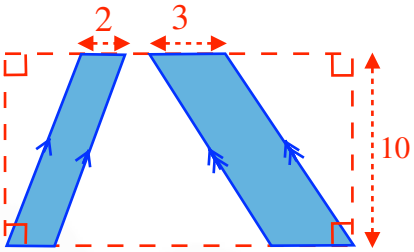
N.B. In quadrilaterals, the ***diagonals*** connect the vertices that are opposite each other.

E.g. 2 Calculate the shaded area of these shapes (all lengths in cm):

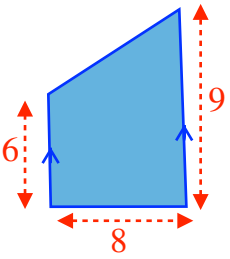
(a)



(b)



(c)



Working: (a) ***The blue shape is not a trapezium so we should calculate the areas of the four triangles and take them away from the area of the rectangle.***

Area of rectangle $= 8 \times 9 = 72$

Area of triangles $= \dfrac{5 \times 2}{2} + \dfrac{6 \times 6}{2} + \dfrac{3 \times 3}{2} + \dfrac{4 \times 5}{2} = 37.5$

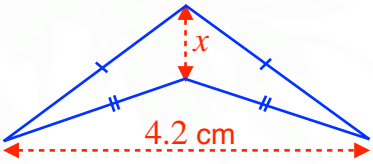
Shaded area $= 72 - 37.5 = 34.5$ cm²

(b) ***Here we should calculate the area of the two parallelograms. The perpendicular height of each parallelogram is*** 10 ***cm.***

Shaded area $= 2 \times 10 + 3 \times 10 = 50$ cm²

(c) Area of trapezium $= \dfrac{1}{2}(a+b)h = \dfrac{1}{2}(6+9) \times 8 = 60$ cm²

E.g. 3 The area of the arrowhead is 6 cm². Find the value of x. Leave your answer as an improper fraction



Working: ***Area of arrowhead*** $= \dfrac{a \times b}{2}$

$$6 = \frac{4.2 \times x}{2}$$

Multiply both side by 2***:*** $12 = 4.2x$

Divide both sides by 4.2***:*** $x = \dfrac{12}{4.2} = \dfrac{20}{7} (\approx 2.86)$ cm.

E.g. 4 The parallel sides of a trapezium of area 105 cm² are 15 cm apart. Given that one of the parallel sides has length 5 cm, calculate the length of the other parallel side.

Working: ***Area of trapezium*** $= \dfrac{1}{2}(a+b)h$***:*** $105 = \dfrac{1}{2}(x+5) \times 15$

Divide both sides by 15***:*** $7 = \dfrac{1}{2}(x+5)$

Multiply both sides by 2***:*** $14 = x + 5$

Subtract 5 ***from both sides:*** $x = 9$

The length of the other parallel side is 9 cm.

E.g. 5 The side of the small square is half the length of the side of the large square. The L-shape has an area of 75 cm^2. Find the length of the side of the large square.

Working: The L-shape has 3 times the area of the small square so the small square has area 25 cm^2.
The area of the large square is 100 cm^2.
The length of the side of the large square is $\sqrt{100} = 10$ cm

E.g. 6 Calculate the radius of a circle with an area equal to the sum of the areas of three circles of radii 2 cm, 3 cm and 4 cm respectively. Give your answer to 3 s.f..

Working: Let r be the radius of the big circle.

$$\pi r^2 = \pi \times 2^2 + \pi \times 3^2 + \pi \times 4^2$$
$$\pi r^2 = 4\pi + 9\pi + 16\pi$$
$$\pi r^2 = 29\pi$$

Divide by π: $$r^2 = 29$$
$$r = \sqrt{29} \approx 5.39$$

The radius of a circle is 5.39 (3 s.f.).

Video: Area of a trapezium
Video: Area of a triangle
Need area of circle

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p424 M13.1 Qu 1-16 Draw all diagrams
A*-G class textbook:	p379 M13.1 Qu 1-15, 17 Draw all diagrams.
9-1 homework book:	p143 M13.1 Qu 1-10 Draw all diagrams
A*-G homework book:	p105 M13.1 Qu 1-10 Draw all diagrams