Calculations with error intervals

Starter

1. **(Review of last lesson)** Write down the error interval of 7200 when it has been:
   (a) rounded to 2 s.f. (b) truncated to 2 s.f.

   Working: (a) Next number rounded to 1 s.f. down is 7100
   Next number rounded to 1 s.f. up is 7300
   The error interval is $7150 \leq x < 7250$

   (b) Lower bound is the number itself i.e. 7200
   Upper bound is the next number up 7300
   The error interval is $7200 \leq x < 7300$

2. Two numbers, x and y, rounded to the nearest whole number become 7 and 4 respectively.
   (a) Write down the error interval for x and the error interval for y.
   (b) State the smallest and largest possible values of $x + y$.
   Write down your calculation.
   (c) State the smallest and largest possible values of $x - y$.
   Write down your calculation.
   (d) State the smallest and largest possible values of $x \times y$.
   Write down your calculation.
   (e) State the smallest and largest possible values of $x \div y$.
   Write down your calculation.

   Working: (a) The error interval for x is $6.5 \leq x < 7.5$.
   The error interval for y is $3.5 \leq x < 4.5$.

   (b) Lower bound = Smallest value $= 6.5 + 3.5 = 10$
   Upper bound = Largest value $= 7.5 + 4.5 = 12$

   (c) Lower bound = Smallest value $= 6.5 - 4.5 = 2$
   Upper bound = Largest value $= 7.5 - 3.5 = 4$

   (d) Lower bound = Smallest value $= 6.5 \times 3.5 = 22.75$
   Upper bound = Largest value $= 7.5 \times 4.5 = 33.75$

   (e) Lower bound = Smallest value $= 6.5 \div 4.5 = 1.\dot{4}$
   Upper bound = Largest value $= 7.5 \div 3.5 = 2.14$ (3 s.f.)

E.g. 1 Using your answers to the starter, copy and complete the table below:

x and y	Lower bound	Upper bound
Addition	$\text{lb}(x) + \text{lb}(y)$	
Subtraction		
Multiplication		
Division		

Working:

x and y	Lower bound	Upper bound
Addition	$\text{lb}(x) + \text{lb}(y)$	$\text{ub}(x) + \text{ub}(y)$
Subtraction	$\text{lb}(x) - \text{ub}(y)$	$\text{ub}(x) - \text{lb}(y)$
Multiplication	$\text{lb}(x) \times \text{lb}(y)$	$\text{ub}(x) \times \text{ub}(y)$
Division	$\text{lb}(x) \div \text{ub}(y)$	$\text{ub}(x) \div \text{lb}(y)$

E.g. 2 If $a = 3.1$ and $b = 8.4$, both correct to 1 d.p., find:

(a) the upper bound of $a + b$ (b) the lower bound of $b - a$

(c) the lower bound of ab (d) the upper bound of $\frac{b}{a}$

Working:

(a) The error interval of $a = 3.1$ is $3.05 \leq a < 3.15$
The error interval of $b = 8.4$ is $8.35 \leq a < 8.45$
Upper bound of $a + b = \text{ub}(a) + \text{ub}(b) = 3.15 + 8.45 = 11.6$

(b) The lower bound of $b - a = \text{lb}(b) - \text{ub}(a) = 8.35 - 3.15 = 5.2$

(c) The lower bound of $ab = \text{lb}(a) \times \text{lb}(b) = 3.05 \times 8.35 = 25.4675$

(d) The upper bound of $\frac{b}{a} = \frac{\text{ub}(b)}{\text{lb}(a)} = \frac{8.45}{3.05} = 2.77$ (3 s.f.)

E.g. 3 The radius of a circle, r, is given as 6.83 to 2 d.p. Calculate the error interval of:

(a) the circumference (b) the area.

Give your answers to 4 s.f..

Working:

(a) The error interval of 6.83 is $6.825 \leq r < 6.835$
Lower bound $= 2 \times \pi \times 6.825 = 42.88$
Upper bound $= 2 \times \pi \times 6.835 = 42.95$
Error interval: $42.88 \leq$ Circumference < 42.95

(b) Lower bound $= \pi \times 6.825^2 = 146.3$
Upper bound $= 2 \times \pi \times 6.835 = 146.8$
Error interval: $146.3 \leq$ Area < 146.7

E.g. 4 Abdul is travelling from Ripon to London. When he set off, his GPS said it was a journey of 230 miles, to the nearest 5 miles. After a few hours the speedometer in his car indicates he has covered 132 miles, to the nearest mile. What is the least distance he has left to travel?

Working:
Error interval of journey distance: $227.5 \leq x < 232.5$
Error interval of distance covered: $131.5 \leq y < 132.5$
Least distance left to cover $= 227.5 - 132.5 = 95$ miles

E.g. 5 A notice in a lift in Brazil says that it can carry 1600 kg safely. This figure is correct to the nearest 100 kg. The weight of the average person in Brazil is 68 kg to the nearest kilogram. What is the maximum number of people that the lift can ***safely*** hold? Give a reason why your value may be too high.

Working:
Error interval of lift: $1550 \leq x < 1625$
Error interval of person's weight: $67.5 \leq y < 68.5$
Think "worst case scenario".
Maximum safe number of people $\equiv$ lower bound
Maximum safe number of people $= \dfrac{\text{lb}(1600)}{\text{ub}(68)} = \dfrac{1550}{68.5} \approx 22.6$
The maximum number of people that the lift can ***safely*** hold is 22.
The value may be too high if 22 people enter the lift who are all above average weight.

E.g. 6 A crane has a cable with a breaking strain of 5300 kg measured to 2 significant figures. It is used to lift crates which weigh 100 kg measured to the nearest 10 kg. What is the greatest number of crates that can be lifted so that the cable will definitely not break?

Working:
Error interval of breaking strain: $5250 \leq x < 5350$
Error interval of mass of crates: $95 \leq y < 105$
Think "worst case scenario".
Maximum safe number of people $\equiv$ lower bound
Maximum safe number of people $= \dfrac{\text{lb}(5300)}{\text{ub}(100)} = \dfrac{5250}{105} = 50$
The greatest number of crates that can be lifted so that the cable will definitely not break is 50.

Video 1: Calculations with error intervals
Video 2: Calculations with error intervals

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p139 E5.1 Qu 1-13
A*-G class textbook:	p130 E5.1 Qu 1-10
9-1 homework book:	p49 E5.1 Qu 1-9
A*-G homework book:	p36 E5.1 Qu 1-6