

Equation of a straight line

Starter

1. **(Review of last lesson)**

Consider the line segment joining the points $(-3, 4)$ and $(2m, n)$.

- (a) Find an expression for the gradient of the line segment in terms of m and n
 (b) Calculate the value of m or n given that:
 (i) the line is parallel to the x -axis,
 (ii) the line is parallel to the y -axis.

Working: (a) **Label the points:** $(-3, 4)$ $(2m, n)$
 (x_1, y_1) (x_2, y_2)

Sub. into formula: Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{n - 4}{2m - -3} = \frac{n - 4}{2m + 3}$

(b) (i) parallel to the x -axis \Rightarrow Gradient = 0
 $\frac{n - 4}{2m + 3} = 0 \Rightarrow n - 4 = 0 \therefore n = 4$

(ii) parallel to the y -axis \Rightarrow Line is vertical
 Since vertical lines have no run $\Rightarrow 2m + 3 = 0$
 $\Rightarrow 2m = -3 \Rightarrow m = -\frac{3}{2} = -1.5$

2. **(Review of previous material)** State the gradient and y -intercept of these lines:

- (a) $y = 4x + 7$ (b) $y = 9 - 5x$ (c) $x + y + 3 = 0$

Working: (a) Gradient = 4 y -intercept = 7
 (b) Gradient = -5 y -intercept = 9
 (c) Rearrange to $y = -x - 3$
 Gradient = -1 y -intercept = -3

E.g. 1 State the gradient and y -intercept of these lines:

- (a) $6x + 3y = 2$ (b) $5y - 6x - 15 = 0$
 (c) $y = 7$ (d) $x = -4$

Working: (a) $6x + 3y = 2$
Subtract $6x$ from both sides: $3y = 2 - 6x$
Divide both side by 3: $y = \frac{2}{3} - 2x$

Gradient = -2 y -intercept = $\frac{2}{3}$

(b) $5y - 6x - 15 = 0$
Add $6x + 15$ to both sides: $5y = 6x + 15$
Divide both side by 5: $y = \frac{6}{5}x + 3$

Gradient = $\frac{6}{5}$ y -intercept = 3

- (c) $y = 7$ is a horizontal line passing through 7
Gradient = 0 y -intercept = 7
- (d) $x = -4$ is a vertical line
The gradient of a vertical line is considered undefined.
The $x = -4$ does not intercept the y -axis so there is no y -intercept.

E.g. 2 Find the equation of the line that has the gradient and passes through the point given:

- (a) gradient = -4 , passes through $(-2, 7)$
 (b) gradient = $\frac{1}{3}$, passes through $(6, -5)$
 (c) gradient = m , passes through (x_1, y_1)

Working: (a) **Substitute the gradient into $y = mx + c$:** $y = -4x + c$
Substitute $(-2, 7)$ into $y = -4x + c$: $7 = -4 \times (-2) + c$
 $7 = 8 + c$
 $c = -1$

The equation of the line is $y = -4x - 1$

(b) **Substitute the gradient into $y = mx + c$:** $y = \frac{1}{3}x + c$
Substitute $(6, -5)$ into $y = \frac{1}{3}x + c$: $-5 = \frac{1}{3} \times 6 + c$
 $-5 = 2 + c$
 $c = -7$

The equation of the line is $y = \frac{1}{3}x - 7$

(c) **Substitute the gradient into $y = mx + c$:** $y = mx + c$
Substitute (x_1, y_1) into $y = mx + c$: $y_1 = m \times x_1 + c$
 $y_1 = mx_1 + c$
 $c = y_1 - mx_1$

Substituting:

$$y = mx + y_1 - mx_1$$

Rearranging:

$$y - y_1 = mx - mx_1$$

Factorising the RHS:

$$y - y_1 = m(x - x_1)$$

The equation of the line is $y - y_1 = m(x - x_1)$.

E.g. 3 Using the formula $y - y_1 = m(x - x_1)$, find the equation of the straight line which has:

- (a) gradient = 2 , passes through $(3, -4)$
 (b) gradient = -5 , passes through $(-1, 8)$

Working: (a) $y - y_1 = m(x - x_1)$: $y - (-4) = 2(x + 3)$
 $y + 4 = 2x + 6$
 $y = 2x - 2$

(b) $y - y_1 = m(x - x_1)$: $y - 8 = -5(x - (-1))$
 $y - 8 = -5(x + 1)$
 $y - 8 = -5x - 5$
 $y = -5x + 3$

E.g. 4 Find the equation of the line passing through

(a) (5, 1) and (2, -5)

(b) (2, -1) and (4, -9)

Working: (a) Label the points: (5, 1) (2, -5)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 1}{2 - 5} = \frac{-6}{-3} = 2$$

Substitute the gradient into $y = mx + c$: $y = 2x + c$

Substitute (5, 1) into $y = 5x + c$: $1 = 2 \times 5 + c$

$$1 = 10 + c$$

$$c = -9$$

The equation of the line is $y = 2x - 9$

(b) Label the points: (2, -1) (4, -9)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9 - -1}{4 - 2} = \frac{-8}{2} = -4$$

Substitute the gradient into $y = mx + c$: $y = -4x + c$

Substitute (2, -1) into $y = -4x + c$: $-1 = (-4) \times 2 + c$

$$-1 = -8 + c$$

$$c = 7$$

The equation of the line is $y = -4x + 7$

Video: [y = mx + c](#)

Video: [Finding the equation of a straight line](#)

Video: [Finding the equation of a straight through 2 points](#)

[Solutions to Starter and E.g.s](#)

Exercise

9-1 class textbook:

p196 M6.14 Qu 1-20 odd, 25, 27-36, 39

A*-G class textbook:

p180 M6.12 Qu 1-20 odd, 25, 27-36

9-1 homework book:

p70 M6.14 Qu 1-10

A*-G homework book:

p53 M6.12 Qu 1-8