Expanding triple brackets

Starter

1. (Review of last lesson)

Find the values of a, b and c that turn these statements into identities (i.e. true for all values of x):

(a) $ax^2 + bx + c \equiv (x+3)^2$ (b) $(ax+b)(x-2) \equiv 2x^2 + cx - 10$

Working: (a) $ax^2 + bx + c \equiv (x+3)^2$

The RHS can be expanded to $x^2 + 6x + 9$

So $ax^2 + bx + c \equiv x^2 + 6x + 9$

Equating coefficients of x^2: $a = 1$

Equating coefficients of x: $b = 6$

Equating the constant term: $c = 9$

(b) $(ax+b)(x-2) \equiv 2x^2 + cx - 10$

Expand the LHS: $ax^2 - 2ax + bx - 2b = ax^2 + (b-2a)x - 2b$

So $ax^2 + (b-2a)x - 2b \equiv 2x^2 + cx - 10$

Equating coefficients of x^2: $a = 2$

Equating coefficients of x: $b - 2a = c$

Equating the constant term: $-2b = 10$ $\therefore b = 5$

So $c = b - 2a = 5 - 2 \times 2 = 1$

$a = 2$, $b = 5$ and $c = 1$

2. (Review of last lesson)

Write the '=' or the '≡' symbol in the box to make each statement mathematically correct.

(a) $x^2 + 6x + 5 \square (x+1)(x+5)$

(b) $x^2 + 6x + 5 \square (x+1)(x+4)$

Working: (a) $x^2 + 6x + 5 \square (x+1)(x+5)$

Expanding the RHS gives $x^2 + 5x + x + 5 = x^2 + 6x + 5$

Since the expression on the LHS is the same as the expression on the RHS, it is true for all values of x. Hence, we need the '≡' symbol.

(b) $x^2 + 6x + 5 \square (x+1)(x+4)$

Expanding the RHS gives $x^2 + 4x + x + 4 = x^2 + 5x + 4$

Since the expression on the LHS is ***not*** the same as the expression on the RHS, it is ***not*** true for all values of x. Hence, we need the '=' symbol.

3. (a) Expand and simplify $(x+2)(x+5)$.

(b) Hence expand and simplify $(x+4)(x+2)(x+5)$.

Working: (a) $(x+2)(x+5) = x^2 + 5x + 2x + 10 = x^2 + 7x + 10$

(b) $(x+4)(x+2)(x+5) = (x+4)(x^2 + 7x + 10)$

$= x^3 + 7x^2 + 10x + 4x^2 + 28x + 40$

$= x^3 + 11x^2 + 38x + 40$

E.g. 1 Expand and simplify:

(a) $(x+1)(x+3)(x+4)$

(b) $(x-1)(x+2)(x+4)$

(c) $(2x+3)(x-3)(x+3)$

(d) $(x-2)(x+5)^2$

(e) $(3x-4)(x-1)^2$

(f) $(x+4)^3$

Working:

(a) $(x+3)(x+4) = x^2+4x+3x+12 = x^2+7x+12$

$\therefore (x+1)(x+3)(x+4) = (x+1)(x^2+7x+12)$

$= x^3+7x^2+12x+x^2+7x+12$

$= x^3+8x^2+19x+12$

(b) $(x+2)(x+4) = x^2+4x+2x+8 = x^2+6x+8$

$\therefore (x-1)(x+2)(x+4) = (x-1)(x^2+6x+8)$

$= x^3+6x^2+8x-x^2-6x-8$

$= x^3+5x^2+2x-8$

(c) $(x-3)(x+3) = x^2+3x-3x-9 = x^2-9$

$\therefore (2x+3)(x-3)(x+3) = (2x+3)(x^2-9)$

$= 2x^3-18x+3x^2-27$

$= 2x^3+3x^2-18x-27$

(d) $(x+5)^2 = x^2+10x+25$

$\therefore (x-2)(x+5)^2 = (x-2)(x^2+10x+25)$

$= x^3+10x^2+25x-2x^2-20x-50$

$= x^3+8x^2+5x-50$

(e) $(x-1)^2 = x^2-2x+1$

$\therefore (3x-4)(x-1)^2 = (3x-4)(x^2-2x+1)$

$= 3x^3-6x^2+3x-4x^2+8x-4$

$= 3x^3-10x^2+11x-4$

(f) $(x+4)^2 = x^2+8x+16$

$\therefore (x+4)^3 = (x+4)(x^2+8x+16)$

$= x^3+8x^2+16x+4x^2+32x+64$

$= x^3+12x^2+48x+64$

Video: **Expanding triple brackets**

Solutions to Starter and E.g.s

Exercise

9-1 class textbook: p106 E4.1 Qu 1-8, 9-10*, 11-19, 20*
A*-G class textbook: No exercise
9-1 homework book: p37 E4.1 Qu 1-10
A*-G homework book: No exercise