

## Linear inequalities

### Starter

1. (Review of last lesson)

When a number is doubled and then added to 67, the result is 213. Find the number.

**Working:** A number is doubled...  $\Rightarrow 2x$   
 ... and then added to 67...  $\Rightarrow 2x + 67$   
 ...the result is 213...  $\Rightarrow 2x + 67 = 213$   
*Subtract 6 from each side:*  $2x = 207$   
*Divide both sides by 2:*  $x = 103.5$   
 The number is 103.5.

2. (Review of last lesson)

Two angles of an isosceles triangle are  $a$  and  $a + 10$ . Find two possible values of  $a$ .

**Working:** Either  $a$  or  $a + 10$  is the angle that appears twice in the isosceles triangle.

*$a$  appears twice:*  $a + a + a + 10 = 180$   
*Collect like terms:*  $3a + 10 = 180$   
*Subtract 10 from each side:*  $3a = 170$   
*Divide both sides by 3:*  $a = \frac{170}{3} = 56\frac{2}{3}$

...and...

*$a + 10$  appears twice:*  $a + a + 10 + a + 10 = 180$   
*Collect like terms:*  $3a + 20 = 180$   
*Subtract 10 from each side:*  $3a = 160$   
*Divide both sides by 3:*  $a = \frac{160}{3} = 53\frac{1}{3}$

The two possible values of  $a$  are  $56\frac{2}{3}$  and  $53\frac{1}{3}$ .

3. (a) Solve the equation  $2x - 1 = 5$ .  
 (b) Using similar working to part (a), find the range of values of  $x$  that satisfy  $2x - 1 \geq 5$ .

**Working:** (a)  $2x - 1 = 5$   
*Add 1 to both sides:*  $2x = 6$   
*Divide both sides by 2:*  $x = 3$

(b)  $2x - 1 \geq 5$   
*Add 1 to both sides:*  $2x \geq 6$   
*Divide both sides by 2:*  $x \geq 3$

- E.g. 1** (a) Solve the equation  $7 - 2x = 11$ .  
 (b) Find the range of values of  $x$  that satisfy  $7 - 2x < 11$  using the following methods:  
 (i) by moving the the  $-2x$  to the other side.  
 (ii) by keeping the  $-2x$  on the left-hand side.

**Working:** (a) **One method could be:**

$$\begin{array}{l}
 7 - 2x = 11 \\
 \text{Add } 2x \text{ to both sides:} \quad 7 = 11 + 2x \\
 \text{Subtract 11 from both sides:} \quad -4 = 2x \\
 \text{Divide both sides by 2:} \quad -2 = x \\
 \text{Make sure the unknown is on the LHS:} \quad x = -2
 \end{array}$$

(b) (i)

$$\begin{array}{l}
 7 - 2x < 11 \\
 \text{Add } 2x \text{ to both sides:} \quad 7 < 11 + 2x \\
 \text{Subtract 11 from both sides:} \quad -4 < 2x \\
 \text{Divide both sides by 2:} \quad -2 < x \\
 \text{Make sure the unknown is on the LHS:} \quad x > -2
 \end{array}$$

(ii)

$$\begin{array}{l}
 7 - 2x < 11 \\
 \text{Subtract 11 from both sides:} \quad -2x < 4 \\
 \text{Divide both sides by } -2: \quad x < -2
 \end{array}$$

**But this contradicts the answer from (i)!**  
**Which one is correct? Let us investigate.**

**E.g. 2** Without moving the  $x$ , find the range of values of  $x$  that satisfy:

(a)  $-2x < 8$  (b)  $-\frac{x}{3} > -5$

**Working:** (a)

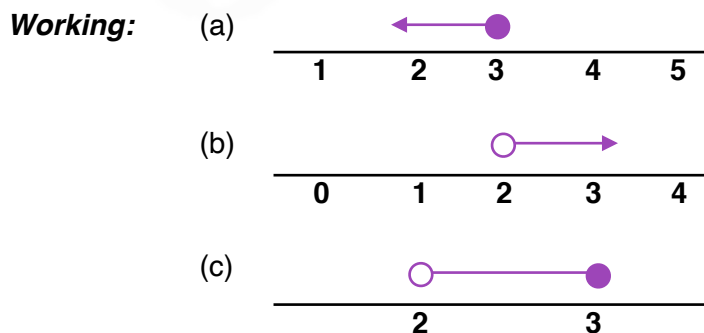
$$\begin{array}{l}
 -2x < 8 \\
 \text{Divide by } -2, \text{ change direction of inequality:} \\
 x > \frac{8}{-2} \\
 x > -4
 \end{array}$$

(b)

$$\begin{array}{l}
 -\frac{x}{3} > -5 \\
 \text{Multiply by } -2, \text{ change direction of inequality:} \\
 x < -3 \times -5 \\
 x < 15
 \end{array}$$

**E.g. 3** Express these inequalities diagrammatically:

(a)  $x \leq 3$  (b)  $x > 2$  (c)  $2 < x \leq 3$

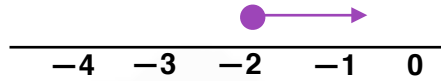


**E.g. 4** Solve these inequalities, expressing your answer as both an inequality and is diagram form:

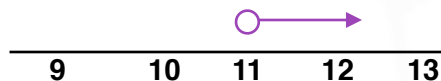
(a)  $-3x - 7 \leq -1$

(b)  $65 - 7x < -12$

**Working:** (a)  $-3x - 7 \leq -1$   
 Add 7:  $-3x \leq 6$   
 Divide by  $-2$ :  $x \geq -2$  *direction of inequality changes*



(b)  $65 - 7x < -12$   
 $-7x < -77$   
 $x > 11$  *direction of inequality changes*



**E.g. 5** Solve the inequalities:

(a)  $8 < 3x - 10 < 23$

(b)  $17 < 6x + 5 \leq 29$

(c)  $7 < 15 - 4x < 39$

(d)  $x - 19 \leq 5x - 3 < x + 5$

**Working:** (a)  $8 < 3x - 10 < 23$   
 $8 < 3x - 10$        $3x - 10 < 23$       *2 separate inequalities*  
 $18 < 3x$                $3x < 33$       *solve separately*  
 $6 < x$                    $x < 11$   
*Combine into 1 inequality:*  $6 < x < 11$

(b)  $17 < 6x + 5 \leq 29$   
 $17 < 6x + 5$        $6x + 5 \leq 29$       *2 separate inequalities*  
 $12 < 6x$                $6x \leq 24$       *solve separately*  
 $2 < x$                    $x \leq 4$   
*Combine into 1 inequality:*  $2 < x \leq 4$

(c)  $7 < 15 - 4x < 39$   
 $7 < 15 - 4x$        $15 - 4x < 39$       *2 separate inequalities*  
 $-8 < -4x$                $-4x < 24$       *solve separately*  
 $2 > x$                    $x > -6$   
*Combine into 1 inequality:*  $-6 < x < 2$   
**N.B.**  $2 > x > -6$  would be incorrect.

(d)  $x - 19 \leq 5x - 3 < x + 5$   
 $x - 19 \leq 5x - 3$        $5x - 3 < x + 5$   
 $-16 \leq 4x$                $4x < 8$   
 $-4 \leq x$                    $x < 2$   
*Combine into 1 inequality:*  $-4 \leq x < 2$

**Video:** [Solving inequalities with one sign](#)  
**Video:** [Solving inequalities \(two signs\)](#)  
[Solutions to Starter and E.g.s](#)

**Exercise**

9-1 class textbook:

p509 M16.1 Qu 1-7 (may need to explain Qu 5)

A\*-G class textbook:

p465 M16.1 Qu 1-6 (may need to explain Qu 4)

9-1 homework book:

p172 M16.1 Qu 1-4

A\*-G homework book:

p128 M16.1 Qu 1-6