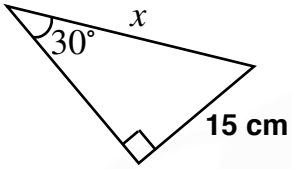
Mixed trigonometry problems

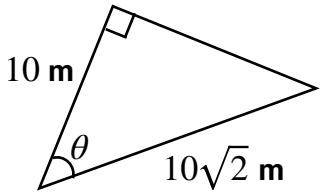
Starter

1. **(Review of last lesson)** Without using a calculator, find the marked length or angle in these right-angled triangles, giving your answers exactly and rationalising surds where needed:

(a)



(b)



Working:

(a) opp and hyp $\Rightarrow$ sin

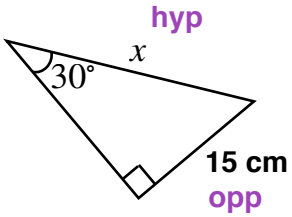
$\sin\theta = \frac{\text{opp}}{\text{hyp}}$: $\quad \sin 30 = \frac{15}{x}$

$x = \frac{15}{\sin 30}$

But $\sin 30 = \frac{1}{2}$: $\quad x = \frac{15}{\frac{1}{2}}$

$x = 15 \times \frac{2}{1}$

$x = 30$ cm



(b) adj and hyp $\Rightarrow$ cos

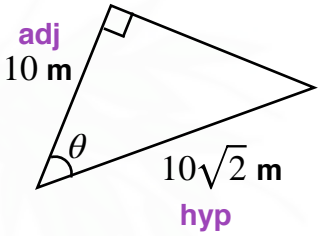
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$: $\quad \cos\theta = \frac{10}{10\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\cos\theta = \frac{\sqrt{2}}{2}$

But $\cos 45 = \frac{\sqrt{2}}{2}$ $\Rightarrow$ $\theta = 45°$



E.g. 1 A ladder of length 5 m stands against a wall. It makes an angle of $65°$ with the horizontal. How far up the wall does it reach?

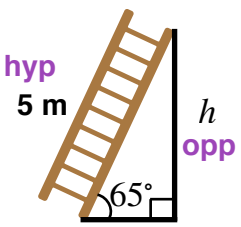
Working: Let h be how far the ladder reaches up the wall.

opp and hyp $\Rightarrow$ sin

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$: $\quad \sin 65 = \frac{h}{5}$

$5 \sin 65 = x$

$x = 4.53$ cm (3 s.f.)



E.g. 2 The base of a rectangle is 8 cm long. The diagonal from top to bottom makes an angle of $74°$ with the horizontal base. Calculate the area of the rectangle to 1 d.p..

Working: Let x be the height of the rectangle

opp and adj $\Rightarrow$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$:

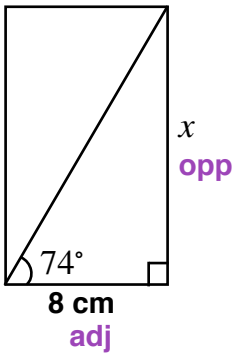
$$\tan 74 = \frac{x}{8}$$

$$8\tan 74 = x$$

N.B. *Do not round half-way through the calculation.*

$$\text{Area} = 8 \times 8\tan 74 = 64\tan 74 = 223.2$$

The area of the rectangle is 223.2 cm².



E.g. 3 (a) Calculate the size of one interior angle of a regular pentagon.

Hint: Sum of interior angles $= 180(n-2)$

(b) Find the shortest distance from the centre of a regular pentagon of side 6 cm, to one of the sides. Give your answer exactly.

(c) Hence find the area of the regular pentagon.

Working: (a) Sum of interior angles $= 180(n-2)$

$$= 180(5-2)$$

$$= 540°$$

$$\text{Size of one interior angle} = \frac{540°}{5}$$

$$= 108°$$

(b) opp and adj $\Rightarrow$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$:

$$\tan 54 = \frac{x}{3}$$

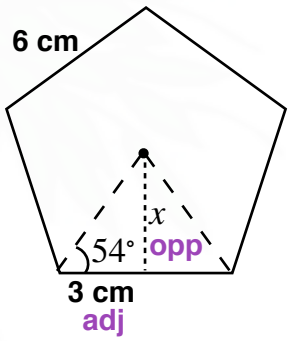
$$3\tan 54 = x$$

The shortest distance is $3\tan 54°$ cm

(c) $$\text{Area of one triangle} = \frac{6 \times 3\tan 54}{2}$$

$$= 9\tan 54$$

The area of pentagon is $5 \times 9\tan 54 = 61.9$ cm².



Video: https://www.youtube.com/watch?v=uVSGy_no7ul

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p327 M10.11 Qu Qu 1-23 odd
A*-G class textbook:	p290 M10.10 Qu 1-25 odd
9-1 homework book:	p114 M10.11 Qu 1-12
A*-G homework book:	p83 M10.10 Qu 1-11 (Qu 11 requires circle theorems)