Proof with Angles

Starter

1. (Review of last lesson)
The exterior angle of a regular polygon is $18°$. How many sides does the polygon have?

Working: *Each exterior angle* $= \frac{360°}{n}$: $\quad 18° = \frac{360°}{n}$

$$n = \frac{360°}{18°} = 20 \text{ sides}$$

The polygon has 20 sides.

2. (Review of last lesson)
A regular polygon has interior angles of $150°$. How many sides does the polygon have?

Working: (a) Exterior angle $= 180° - 150° = 30°$

Each exterior angle $= \frac{360°}{n}$: $\quad 30° = \frac{360°}{n}$

$$n = \frac{360°}{40°} = 9 \text{ sides}$$

3. (Review of last lesson) Tasneem claims she has found a regular polygon whose interior angle is 80° more than the exterior angle. If the regular polygon exists, how many sides does it have?

Working: Let the size of an exterior angle be x.
The size of an interior angle is $x + 80°$.
Exterior angle $+$ Interior angle $= 180°$:

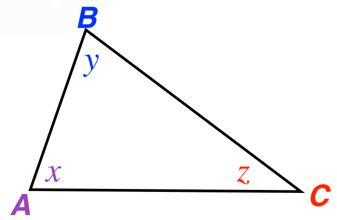
$$x + x + 80° = 180°$$
$$2x = 100°$$
$$x = 50°$$

Each exterior angle $= \frac{360°}{n}$: $\quad 50° = \frac{360°}{n}$

$$n = \frac{360°}{50°} = 7.2 \text{ sides}$$

A regular polygon does not exist since the number of sides must be an integer.

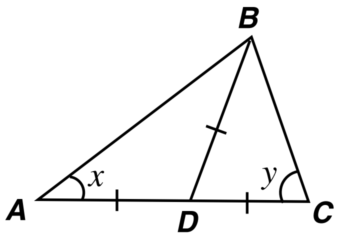
E.g. 1 Give the different angle notations for angles:
(a) y (b) z.



Working: (a) $y \equiv \angle ABC$ or $\angle CBA$ or $A\hat{B}C$ or $C\hat{B}A$ or $\hat{B}$

(b) $z \equiv \angle BCA$ or $\angle ACB$ or $B\hat{C}A$ or $A\hat{C}B$ or $\hat{C}$

E.g. 2 Prove that $x + y = 90^\circ$.

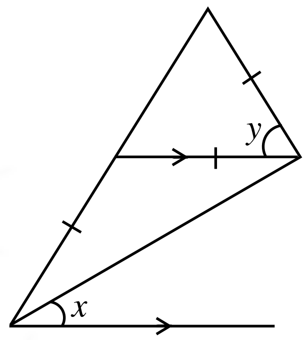


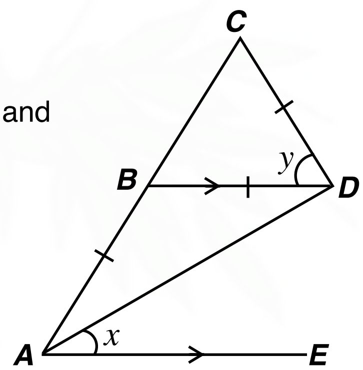
Working:

$\angle BAD = x \Rightarrow \angle ABD = x$ ($\triangle ABD$ is isosceles)
$\angle ADB = 180 - 2x$ since angles in a triangle add up to 180°
$\angle BCD = y \Rightarrow \angle CBD = y$ ($\triangle BCD$ is isosceles)
$\angle BDC = 180 - 2y$ since angles in a triangle add up to 180°
$\angle ADB + \angle BDC = 180^\circ$ since angles on a straight line add up to 180°
So $180^\circ - 2x + 180^\circ - 2y = 180^\circ$ $\quad 180^\circ = 2x + 2y$
Divide both sides by 2: $\quad x + y = 90^\circ$

E.g. 3 Prove that $4x + y = 180^\circ$.

Hint: Label the important points of the shape with a capital letter.



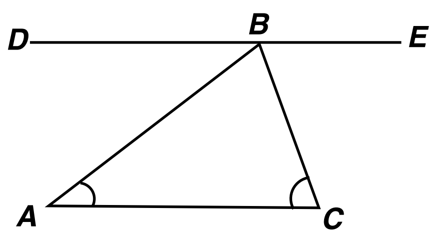


Working:

$C\hat{B}D = \dfrac{180^\circ - y}{2}$ since $\triangle BCD$ is isosceles and angles in a triangle add up to 180°
$D\hat{A}E = A\hat{D}B = x$ (alternate angles)
$A\hat{D}B = D\hat{A}B = x$ ($\triangle ABD$ is isosceles)
$A\hat{B}D = 180^\circ - 2x$ (angles in a triangle add up to 180°)

$\angle ABD + \angle CBD = 180^\circ$ since angles on a straight line add up to 180°
So $180^\circ - 2x + \dfrac{180^\circ - y}{2} = 180^\circ$
Multiply both sides by 2: $\quad 360^\circ - 4x + 180^\circ - y = 360^\circ$
$\therefore 4x + y = 180^\circ$

E.g. 4 Use the fact that the sum of the angles in a triangle add up to 180°, to prove that the degrees on a straight line are also 180°.



Working:

$B\hat{A}C + A\hat{B}C + A\hat{C}B = 180^\circ$ since angles in a triangle add to 180°
$B\hat{A}C = A\hat{B}D$ (alternate angles)
$B\hat{C}A = C\hat{B}E$ (alternate angles)
$\therefore A\hat{B}D + A\hat{B}C + C\hat{B}E = 180^\circ$
i.e. the sum of the angles in a triangle add up to 180°

Video: Proof with angles

Solutions to Starter and E.g.s

Exercise

9-1 class textbook:	p71 M3.5 Qu 1-10
A*-G class textbook:	p64 M3.5 Qu 1-17 odd
9-1 homework book:	p23 M3.5 Qu 1-8
A*-G homework book:	p17 M3.5 Qu 1-10